Inverse Energy Transfer by Near-Resonant Interactions with a Damped-Wave Spectrum

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The interaction of long-wavelength anisotropic drift waves with the plasma turbulence of electron density advection is shown to produce the inverse energy transfer that condenses onto zonal modes, despite the expectation of forward transfer on the basis of nonconservation of enstrophy. Wave triads with an unstable wave and two waves of a separate, damped spectrum carry the transfer, provided they satisfy a near-resonance condition dependent on turbulence level and wave number.

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Inverse spectral energy transfer in turbulence drives large-scale flows and fields from smaller scale fluctuations, making it important for magnetic dynamos, the generation of large-scale planetary flows and zonal flows in fusion plasmas. Historically, the direction of spectral energy transfer has been linked to dynamical invariants. In 3D Navier-Stokes turbulence the forward cascade of energy to small scales is associated with energy invariance. In 2D the addition of a second invariant, the enstrophy, leads to an inverse energy cascade [1,2]. Plasma turbulence also drives inverse energy transfer, provided spectral transfer is dominated by enstrophy-conserving 2D vorticity advection [3]. This drives large-scale zonal flows in plasma experiments [4]. However, other nonlinearities also arise in plasmas. Advection of electron density [5] breaks enstrophy conservation, producing forward transfer [6]. This nonlinearity dominates spectral transfer at large scales where zonal flows form, making its effect on zonal flow excitation an issue of considerable importance.

There is growing evidence that the canonical relationship between invariants and spectral energy transfer direction pertains to isotropic motions and can be radically modified when the isotropy of turbulent advection is broken by anisotropic wave motion [7,8]. In 3D rotating turbulence, *three-dimensional motions* give rise to inverse spectral energy transfer [7]. This excites quasi-2D vortical columns, whose anisotropy reflects the anisotropy of the zero-frequency inertial waves induced by rotation. Rotating stratified turbulence has similar properties [8]. The mechanism responsible for the inverse transfer is not understood, although exactly resonant three-wave interactions have been implicated. However, the vanishing of the nonlinear coupling coefficient for exact resonance [9,10], a property of small amplitude expansions where broadening is formally zero (weak turbulence), has impeded analytical progress. Numerical analysis has recently shown that the coupling of near-resonant triads must be present for inverse transfer in 3D rotating and stratified turbulence [11].

In this Letter the long-wavelength interaction of unstable, anisotropic trapped electron mode (TEM) waves with the turbulence of electron density advection is shown to create strong inverse energy transfer where otherwise the energy transfer is forward [5,6]. This system and its behavior are relevant to rotating and rotating stratified turbulence because all belong to an empirical similarity group [12]. Calculational advantages offered by TEM might be exploited to gain insight into how wave anisotropy affects transfer in the other systems. In TEM turbulence the mechanism involves two eigenmodes [12,13], one of which is unstable. It saturates by transfer to a separate, damped eigenmode (i.e., a subcritical spectrum of damped waves). Inverse energy transfer is carried by three-wave interactions that couple a wave of the unstable spectrum to two waves of the subcritical spectrum. Inverse transfer is manifested through rough antisymmetry (sign change) of the energy transfer rate for $k \leftrightarrow k'$, where k and k' belong to the subcritical spectrum. Aspects of the rough antisymmetry appear in the basic equations. However, wave properties reside in linear terms and enter spectral transfer solely through turbulent decorrelation. A self-consistent, strong turbulence closure yields the turbulent decorrelation rate, providing an explicit near-resonance condition for inverse transfer. The condition is violated if the damped-wave frequency induced by coupling to the unstable mode exceeds the linear three-wave frequency mismatch. When it is violated the inverse transfer changes direction and becomes forward.

We study the collisionless limit of a TEM model valid for arbitrary collisionality [12,13]. Previous work described the finite-amplitude-induced damping of zonal modes [12]. Here we describe the dynamics of spectral transfer from small scales to all long-wavelength modes, not just zonal modes. Unlike the simulations of Ref. [12], the immediate proximity of an unstable spectrum is not required. Because growth rates are small relative to wave frequencies, the process is relevant to stable systems [7,8] where the physics of inverse transfer has been elusive. The model is

$$\frac{\partial n}{\partial t} - \nabla\phi \times z \cdot \nabla n + \nu(n - \phi) = -v_D \hat{\alpha} \frac{\partial \phi}{\partial y}, \quad (1)$$

$$\frac{\partial}{\partial t}(1-\nabla^2-\epsilon^{1/2})\phi-\epsilon^{1/2}\nu(n-\phi)+\nabla\phi\times z\cdot\nabla\nabla^2\phi=-v_D(1-\epsilon^{1/2}\hat{\alpha})\frac{\partial\phi}{\partial y}, \quad (2)$$

where $n=\epsilon^{1/2}n_e+\phi$ is an effective density, n_e is the density of trapped electrons, ϕ is the potential, $\epsilon^{1/2}$ is the trapping fraction, ν is the instability-driving detrapping rate, v_D is the diamagnetic drift velocity, $\hat{\alpha}=1+(3/2)\eta_e$, and η_e is the ratio of gradient scale lengths for the density and temperature. Normalizations and the derivation are given in Ref. [13]. In the collisionless regime, valid for $v_D k_y>\nu$, where k_y is a typical wave number in the y direction, the growth rate is proportional to ν and requires $n\neq\phi$ ($n_e\neq 0$). The instability propagates with phase velocity proportional to v_D. The rough antisymmetry of inverse spectral transfer is rooted in wave-number-space parities required by reality: $\omega(-k)=-\omega^*(k)$, $n(-k)=n^*(k)$, and $\phi(-k)=\phi^*(k)$. The nonlinearity of the n equation is electron density advection, yielding forward transfer in isotropic turbulence. The nonlinearity of the ϕ equation is vorticity advection, yielding inverse transfer. For long wavelengths $k\ll(\sqrt{n/\phi})_{\rm rms}$, electron density advection dominates spectral transfer. This is the regime of TEM turbulence in fusion devices, and the regime of interest for comparison with rotating and stratified turbulence, where inverse energy transfer is observed with nonlinearities having forward transfer for isotropic fluctuations. Henceforth, advection of vorticity is neglected.

The model is transformed via the eigenmode decomposition [12,13] to deal with the fact that time derivatives are amplitude-dependent superpositions of all eigenfrequencies, and not generally reducible to the frequency of the unstable mode. The correct approach transforms the field equations to nonlinear equations for the eigenmode amplitudes. Writing the two time-dependent eigenmode amplitudes as β_1 and β_2, their evolution obeys

$$\left[\frac{\partial}{\partial t}+i\omega_j\right]\beta_j=-2\sum_{k'}\sum_{m=1}^{2}(-1)^j C_m(k,k')\beta'_m\beta''_1, \quad (3)$$

where the notation $\beta'_j\equiv\beta_j(k',t)$, $\beta''_j\equiv\beta_j(k-k',t)$, $\beta_j\equiv\beta_j(k,t)$ is adopted for shorthand and also applied to $\omega_j(k)$, the eigenmode frequencies [given in Eqs. (4) and (6) of Ref. [13]]; $C_m(k,k')=-(\mathbf{k}'\times\hat{\mathbf{z}}\cdot\mathbf{k})R_m(k')/[R_1(k)-R_2(k)]$ are nonlinear coupling coefficients. The functions $R_m(k)=[i\omega_m(k)-\alpha_{22}(k)]/\alpha_{21}(k)$ are eigenvector components, and $\alpha_{21}(k)=-\epsilon^{1/2}\nu/(1+k^2-\epsilon^{1/2})$ and $\alpha_{22}(k)=[iv_Dk_y(1-\hat{\alpha}\epsilon^{1/2})+\epsilon^{1/2}\nu]/(1+k^2-\epsilon^{1/2})$ are linear coupling coefficients of the original model. It has been assumed that $\beta_2<\beta_1$, a condition that holds throughout the linear growth phase and saturated state [13].

To examine spectral transfer, moments of the β_j equation are taken, yielding evolution equations for $|\beta_1|^2$, $|\beta_2|^2$, and $\langle\beta_1^*\beta_2\rangle$. These are components of the spectral energy density $E(k)=[1+k^2-\epsilon^{1/2}]|\beta_1+\beta_2|^2+\epsilon^{1/2}|R_1(k)\beta_1+R_2(k)\beta_2|^2$. Taking moments generates a hierarchy. The first two equations evolve spectral energy density components and triplet correlations:

$$\left[\frac{\partial}{\partial t}+i\omega_j-i\omega_l^*\right]\langle\beta_j\beta_l^*\rangle=-2\sum_{k'}\sum_{m=1}^{2}[T_{mjl}(k,k')+T^*_{mlj}(k,k')], \quad (4)$$

$$\left[\frac{\partial}{\partial t}+i\omega'_m+i\omega''_1-i\omega_l^*\right]\langle\beta'_m\beta''_1\beta_l^*\rangle=-2\sum_{k'''}\sum_{j=1}^{2}[Q_{mj1l}(k',k''',k)+Q_{1jml}(k-k',k''',k)+Q^*_{ljm1}(k,k''',k-k')], \quad (5)$$

where $T_{mjl}(k,k')=(-1)^jC_m(k,k')\langle\beta'_m\beta''_1\beta^*_l\rangle$ and $Q_{mjnl}(k',k''',k)=(-1)^mC_j(k',k''')\langle\beta_j(k''')\beta_1(k'-k''')\beta_n(k-k')\times\beta_l^*(k)\rangle$ are spectral transfer rates for energy and triplet correlations. Appropriate restrictions on triad summations of T_{mjl} and T^*_{mlj} in Eq. (4) yield the net flux across boundaries separating small and large wave numbers [1]. Hence the transfer direction follows from averages of T_{mjl} over triad interactions. Here a different approach is pursued by identifying triads with manifestly inverse transfer as specified by (i) $\mathrm{Sgn}T_{mjl}(k,k')=-\mathrm{Sgn}T_{mjl}(k',k)$, subject to $\langle\beta_m(k')\beta_1(k-k')\beta_j^*(k)\rangle=\pm\langle\beta_m(k)\beta_1(k'-k)\beta_j^*(k')\rangle$, (ii) $T_{mjl}(k,k')<0$ for $k<k'$, and (iii) $T_{mjl}(k,k')\sim i[\omega_j-\omega_l^*]\langle\beta_j\beta_l^*\rangle$ for $\nu\ll v_Dk_y$. Condition (i) represents rough antisymmetry. It selects antisymmetric transfer (conservative energy exchange between k and k') and the transfer of a less stringent class of three-wave interactions for which the sign and magnitude of T_{mjl} change under $k\leftrightarrow k'$. (For these, the net energy exchanged across the spectrum is not zero; hence additional nonsymmetric three-wave interactions maintain energy conservation.) Because the underlying three-wave correlations must be symmetric or antisymmetric, this condition is intrinsic to the coupling and does not depend on dynamical properties like fluctuation level. Condition (ii) selects inverse transfer; i.e., energy must be removed from $k>k'$ and deposited in $k<k'$. Condition (iii) selects transfer rates that enter in the leading-order saturation balance under the collisionless asymptotic limit ($\nu\ll v_Dk_y$). Conditions (ii) and (iii) are dynamical. They depend on the spectra and turbulent decorrelation rate. Unlike condition (i), a closure is generally required to determine when they are satisfied.

First consider condition (i). The wave-number parity of fluctuations carries over to the eigenmode amplitudes, so that $\beta_j(-k)=\beta_j^*(k)$. Consequently, $\langle\beta_m(k')\beta_1(k-k')\beta_j^*(k)\rangle=\langle\beta_m^*(k')\beta_1(k'-k)\beta_j(k)\rangle^*=\langle\beta_m(k)\beta_1(k'-k)\beta_j^*(k')\rangle^*$, where the last equality holds if $m=j$. Therefore, the real parts of $\langle\beta'_1\beta''_1\beta^*_1\rangle$ and $\langle\beta'_2\beta''_1\beta^*_2\rangle$ are symmetric, and the imaginary parts are antisymmetric under $k \leftrightarrow k'$. This means that T_{111}, T_{121}, T_{222}, and T_{212} are the only energy transfer rates that can possibly satisfy condition (i). These govern transfer of $|\beta_1|^2$ (T_{111}), $\langle\beta_1^*\beta_2\rangle$ (T_{121} and T_{212}), and $|\beta_2|^2$ (T_{222}). To simplify analysis it is helpful to consider condition (iii), because it shows that T_{121} and T_{212} do not contribute to the leading-order saturation balance. Condition (iii) depends on the magnitude of $\langle\beta_m'\beta_1''\beta_j^*\rangle$, which in turn depends on the three fourth-order correlations of Eq. (5) and the temporal response operator that inverts Eq. (5). A statistical closure of the eddy damped quasinormal Markovian (EDQNM) variety is used to evaluate these quantities and solve Eq. (5). Details are given in Ref. [12]. The turbulent decorrelation is not an *ad hoc* eddy damping but is derived self-consistently. The small parameter $\nu/k_y v_D$ allows the usual recursive definition of the turbulent decorrelation rate to be solved order by order. This procedure correctly predicts a variety of quantities dependent in a nontrivial way on the turbulent decorrelation [13], including a large shift of the $|\beta_2|^2$ frequency spectrum to $k_y v_D$ from its linear peak near 0, the phase locking between β_1^* and β_2, and the complex value of $\langle\beta_1^*\beta_2\rangle$. The smallness of $\nu/k_y v_D$ also allows asymptotic expansion and order-by-order solution of the saturation balances of Eq. (4). The long-wavelength condition $k^2 \ll 1$ is formally incorporated into the ordering with $k^2 = O(\nu/k_y v_D)^2$.

The saturation balances show that $\langle\beta_1^*\beta_2\rangle$ reaches steady state under a balance of driving and dissipation with T_{121} and T_{221}, which do not satisfy condition (i). Similarly $|\beta_1|^2$ reaches steady state under a balance of instability drive with T_{211}; T_{111} is subdominant. Consequently, the Kolmogorov-like forward transfer to shorter-wavelength, viscously damped waves within the unstable eigenmode under T_{111} is weaker than the transfer to the damped-wave eigenmode under the nonsymmetric T_{211}. Unlike $|\beta_1|^2$ and $\langle\beta_1^*\beta_2\rangle$, energy transfer within the $|\beta_2|^2$ equation is dominated by the potentially antisymmetric transfer rate, T_{222}.

To determine if T_{222} is antisymmetric, i.e., if it satisfies all parts of condition (i), the symmetry properties of $C_2(k, k')$ must be considered. Note first that T_{222} is paired with T_{222}^* in Eq. (4). Hence, it is the symmetry of $\mathrm{Re}T_{222}$ that is of interest, requiring a separate analysis of $\mathrm{Re}C_2(k, k')\mathrm{Re}\langle\beta_2'\beta_1''\beta_2^*\rangle$ and $\mathrm{Im}C_2(k, k')\mathrm{Im}\langle\beta_2'\beta_1''\beta_2^*\rangle$. The coupling coefficients $C_j(k, k')$ are complicated because they depend on ω_j. Typically $C_j(k, k')$ changes sign for some portion of the four-dimensional wave-number space spanned by $\mathbf{k}$ and $\mathbf{k}'$, while for the remainder there is no sign change. In lowest order C_2 is simple. It is equal to $(\mathbf{k}' \times \hat{\mathbf{z}} \cdot \mathbf{k})(k_y'/k_y)$, which changes sign for all triads. In the next order C_2 is imaginary and changes sign for $1/4$ of the triads. Combining the coupling coefficients and the three-wave correlations, condition (i) is satisfied by $\mathrm{Re}C_2(k, k')\mathrm{Re}\langle\beta_2'\beta_1''\beta_2^*\rangle$ for all triads, and by $\mathrm{Im}C_2(k, k')\mathrm{Im}\langle\beta_2'\beta_1''\beta_2^*\rangle$ for $3/4$ of the triads. However, when evaluated from closure theory most terms of the latter are smaller than the former by a factor of order $\nu/k_y v_D$.

T_{222} is now evaluated using the closure solution of Eq. (5). In the Markovian (steady state) limit, $[\partial/\partial t + i\omega_2' + i\omega_1'' - i\omega_2^*] \rightarrow [i\omega_2' + i\omega_1'' - i\omega_2^* - \Delta\omega_2' - \Delta\omega_1'' - \Delta\omega_2^*]$, where $\Delta\omega_j$ is the complex nonlinear frequency determined self-consistently from the closure [see Eqs. (35) and (36) of Ref. [12]]. From the saturation balances $\Delta\omega_2 \sim ik_y v_D$, whereas $\Delta\omega_1 \sim \nu$. Likewise, from the eigenfrequencies, $\omega_1 \sim k_y v_D$, while $\omega_2 \sim i\nu$. Therefore, $[i\omega_2' + i\omega_1'' - i\omega_2^* - \Delta\omega_2' - \Delta\omega_1'' - \Delta\omega_2^*] \sim [i\omega_1'' - \Delta\omega_2' - \Delta\omega_2^*]$. Near resonance occurs when the linear wave frequency dominates the nonlinear frequency, i.e., $i\omega_1'' \gg \Delta\omega_2' + \Delta\omega_2^*$, and applies to waves k and k' on the damped-wave eigenmode branch and beat wave $k'' = k - k'$ on the unstable branch. Substituting the complete lowest order expressions for $i\omega_1''$, $\Delta\omega_2'$, and $\Delta\omega_2^*$, the near-resonance condition is

$$\frac{v_D(k_y - k_y')(1 - \epsilon^{1/2}\hat{\alpha})^2}{[1 + (k - k')^2 - \epsilon^{1/2}]} \gg \sum_q \frac{|\beta_1(q)|^2(1 + q^2 - \epsilon^{1/2})}{v_D q_y}[C_2(k', k' - q)C_2(k' - q, k') - C_2(k, k - q)C_2(k - q, k)]. \quad (6)$$

Dimensionally, this condition is $\omega_1 \gg C_2\beta_1$, the analog of the Rhines cutoff in quasigeostrophic turbulence [14]. It describes a wave regime where spectral transfer is small but nonzero, so that spectra evolve on very long time scales. It arises classically at small k and k', where $C_2 \ll 1$, but also if β_1 or the coupling factor in [ ] is small.

Using the normal-statistics ansatz of EDQNM, $\mathrm{Re}T_{222}(k, k')$ is evaluated by writing $Q_{mjnl}(k', k''', k)$ in terms of products of second order correlations $\langle\beta_j\beta_l^*\rangle$. The result is given in Ref. [12], Eq. (33), in the terms proportional to $C_2(k, k')/[i\omega_2' + i\omega_1'' - i\omega_2^* - \Delta\omega_2' - \Delta\omega_1'' - \Delta\omega_2^*]$. In the near-resonant limit for $\nu/k_y v_D \ll 1$, eight of the nine terms enter the leading-order energy transfer rate. The resulting expression is complicated, with components producing both forward and inverse transfer, depending on spectrum shape. Inverse transfer dominates for a broad class of physically important spectra. Consider decaying spectra such that $|\beta_1|^2$, $|\beta_2|^2$, and $\langle\beta_1^*\beta_2\rangle$ have the same shape and fall off faster than k^{-1}. The spectra generated in simulations of Eqs. (1) and (2) belong to this class [13], as do the spectra of the canonical balance defining the wave-dominated regime [14], which go as $|\beta_1|^2 \sim |\beta_2|^2 \sim \langle\beta_1^*\beta_2\rangle \sim k^{-2}$. If the transfer range is broad, such that, typically, $\langle\beta_i^*\beta_j\rangle \gg \langle\beta_i'^*\beta_j'\rangle$ for $k < k'$, the expression simplifies to

$$2\mathrm{Re}T_{222}(k,k') = -\sum_{k'} \frac{(\mathbf{k}'\times\hat{\mathbf{z}}\cdot\mathbf{k})^2(1-\epsilon^{1/2})\nu}{v_D^2(1-\epsilon^{1/2}\hat{\alpha})^3}\Bigg[\frac{\nu\epsilon^{1/2}(\hat{\alpha}-1)(1-\epsilon^{1/2})}{v_D k_y^2(k_y-k_y')^2(1-\epsilon^{1/2}\hat{\alpha})^2}|\beta_1''|^2[(k_y'-2k_y)\mathrm{Im}\langle\beta_1^*\beta_2\rangle + (2k_y' - k_y)\mathrm{Im}\langle\beta_1'^*\beta_2'\rangle] + (k_y'\mathrm{Im}\langle\beta_1'^*\beta_2'\rangle - k_y\mathrm{Im}\langle\beta_1^*\beta_2\rangle)\frac{\mathrm{Im}\langle\beta_1''^*\beta_2''\rangle}{k_y^3 k_y'}[J_1(k_y-k_y') + J_2(k_y+k_y')]\Bigg], \tag{7}$$

where $J_1 = -[1+(1/2)\hat{\alpha}\epsilon^{1/2}(1+\epsilon^{1/2}) - 2\epsilon^{1/2}]$ and $J_2 = (1/2)\hat{\alpha}\epsilon^{1/2}(1-\epsilon^{1/2})$.

With one exception, all terms of Eq. (7) explicitly satisfy rough antisymmetry and produce inverse transfer; i.e., they are negative for $k < k'$ and positive for $k > k'$. The exception is the term proportional to J_2, which is the only contribution to leading-order transfer from the 25% of triads in $\mathrm{Im}C_2(k,k')\mathrm{Im}\langle\beta_2'\beta_1''\beta_2^*\rangle$ that do not have rough antisymmetry. This term is roughly symmetric and negative for all k. Hence it constitutes a source in β_2, fed by unstable β_1 modes. (The dominant component of T_{122} also yields energy transfer from $|\beta_1|^2$ to $|\beta_2|^2$.) The evaluation of signs in Eq. (7) relies on simulation results, which show that $\mathrm{Im}\langle\beta_1^*\beta_2\rangle$ and $\mathrm{Re}\langle\beta_1^*\beta_2\rangle$ are stationary and positive [13]. The inverse transfer of T_{222} occurs between damped-spectrum waves at k and k' excited by coupling to an unstable wave at $k - k'$ and is the dominant transfer within the subcritical spectrum for spectra decaying faster than k^{-1}. It requires anisotropic wave motion, as is apparent from its dependence on k_y and k_y'.

Because it is proportional to $(i\omega_1'' - \Delta\omega_2' - \Delta\omega_2^*)^{-1}$, T_{222} changes when the near-resonance condition is violated and $|\Delta\omega_2' + \Delta\omega_2^*| > \omega_1''$. shows tdhat $-\Delta\omega_2' - \Delta\omega_2^*$ has the opposite sign of $i\omega_1''$; hence the direction of transfer reverses. To see this, refer to $\Delta\omega_2$ as given in Eq. (36) of Ref. [13]. In the limit $\nu/k_y v_D \ll 1$ the last term (proportional to $C_2^2|\beta_1''|^2$) dominates. Performing a Taylor expansion of the turbulent response for $k_y < k_y'$, and transforming the k_y' summation to the positive domain, the lowest order contribution is

$$\mathrm{Im}\Delta\omega_2 = -k_y v_D \sum_{k'=0}^{\infty} \frac{2(k_y'k_x)^2|\beta_1'|^2(1-\epsilon^{1/2}\hat{\alpha})}{[\mathrm{Re}\omega_1'^{(0)} + \mathrm{Im}\Delta\omega_2'^{(0)}]^2(1-\epsilon^{1/2})}. \tag{8}$$

For $k_y > 0$, $\mathrm{Im}\Delta\omega_2 < 0$, while $\omega_1 = k_y v_D(1-\epsilon^{1/2}\hat{\alpha})/(1-\epsilon^{1/2}) > 0$. The 180° phase shift is consistent with the observed stationarity of $\langle\beta_1^*\beta_2\rangle$ and the harmonic forcing of an oscillator above its natural frequency. Here, mode coupling drives β_2 at frequencies $\omega_1 \sim k_y v_D \gg \omega_2 \sim 0$. Hence, the inverse transfer of T_{222} reverses and becomes forward when the near-resonance condition is violated.

This Letter has shown that anisotropic TEM waves interact with plasma turbulence to produce inverse energy transfer, even though the nonlinearity of electron density advection that dominates at long wavelengths produces forward energy transfer for isotropic motions. This is significant because it demonstrates that zonal flows can be driven by shorter wavelength unstable fluctuations in fusion plasmas. Because the inverse energy transfer is mediated by the wave frequency and not instability growth rates, energy can move across a linearly stable range to reach very low wave numbers. There it preferentially reaches the zonal wave number $k_y = 0$ under anisotropic spectral transfer [12]. The damped-wave spectrum carries the inverse transfer and saturates the instability.

Inverse transfer is roughly antisymmetric under $k \leftrightarrow k'$. The triads involved must satisfy an explicit near-resonant condition that depends on the turbulence level. Because this process is driven by anisotropic wave frequencies that are 1 order larger than instability growth rates, it is likely to be relevant to inverse energy transfer in rotating and stratified turbulence, even though the waves of the latter systems are neutrally stable, while TEM waves are unstable. The techniques employed here may be useful in identifying the near-resonant condition believed to characterize inverse transfer in those systems. It is possible that an additional member of the similarity group mentioned in the introduction is MHD turbulence, because its anisotropy, $k_\| \ll k_\perp$, is consistent with the zero-frequency shear Alfvén wave $\omega = v_A k_\|$. The work presented here may offer inroads into analysis and characterization of the MHD anisotropy, the role of the anisotropy on turbulent decorrelation, and the nature of inverse transfer processes associated with dynamos in MHD.

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