Plasma confinement by circularly polarized electromagnetic field in toroidal geometry

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A novel plasma confinement concept based on plasma confinement by electromagnetic pressure of circularly polarized electromagnetic fields is proposed. Practical implementation of this concept in a toroidal device is suggested. In this concept the confining field frequency is in the lower range such that the size of the device is much smaller than the vacuum wavelength. Most of the previous radio-frequency (rf) confinement concepts of unmagnetized plasma were related to confinement in rf cavities which operated at high frequency for which the size of the cavity is comparable to the wavelength. Operation at lower frequencies simplifies rf design, reduces Ohmic losses in the conducting walls and probably makes application of superconductors for wall materials more feasible. It is demonstrated that circular (or nearly circular) polarization of the electromagnetic field is required for confinement from both the equilibrium and stability considerations. Numerical analysis of plasma confinement for magnetohydrodynamic plasma model in two-dimensional toroidal geometry is performed. Within this model plasma is confined by the applied rf fields and its equilibrium is stable. Technically feasible compact and medium size toroidal plasma confinement devices based on this concept are proposed. Application of this approach to the fusion reactor requires use of superconducting materials for the toroidal shell to reduce the Ohmic losses. Further theoretical and experimental studies are required for a more reliable conclusion about the attractiveness of this plasma confinement concept. © *2007 American Institute of Physics*.
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I. INTRODUCTION

Plasma confinement by the pressure of the electromagnetic field can be an attractive alternative to other confinement methods (e.g., magnetic or inertial). The confinement is steady state and there is the possibility of using the same field for plasma confinement, heating, and stabilization. The basic principle used in electromagnetic confinement is that the radiation is largely reflected from plasma if the field frequency is less than the electron plasma frequency $\omega < \omega_{pe}$. The reflected radiation exerts pressure on the plasma surface. The radio-frequency (rf) field exerts an active force on plasma particles in the sense that when a particle is shifted toward the boundary rf force pushes it back. In this sense magnetic confinement is passive; there is no restoring force on a shifted (due to collisions, etc.) particle.

The rf ponderomotive force is used in magnetized plasma for stabilization of magnetohydrodynamic (MHD) instabilities. RF stabilization has been investigated for interchange instabilities in mirror machines,[1–4] external kinks in tokamaks,[5,6] resistive wall modes in reversed field pinches,[7] and Rayleigh-Taylor instabilities in liquid metals (see, e.g., Ref. 8).

The direct use of electromagnetic pressure to confine unmagnetized plasma (in steady state) is studied to a much lesser extent. Most of the previous studies of confinement of unmagnetized plasma by the rf field are related to confinement in a resonant cavity.[9–14] In this approach the size of the cavity is comparable to the vacuum wavelength in order to excite cavity eigenmodes. In order to satisfy this condition the field frequency in such configuration is $f \gtrsim 100$ MHz. Ohmic losses in a conducting wall are proportional to $\sqrt{\omega}$. The conclusions of early publications (late 1950s) on this approach to fusion power where that it is unpromising because the Ohmic energy losses in cavity walls would be huge compared to all other energies involved and that the rf electric field amplitude would be impractically large (exceeding 10^6 V/cm). As stated in Ref. 14, the use of high-temperature superconductors for wall materials may significantly improve the feasibility of this rf confinement approach and further study is required to evaluate its attractiveness. The scheme of a possible fusion reactor using superconducting materials is suggested in Ref. 15. This approach also requires a very high Q of the cavity (in excess of 10^9). But the unresolved question remains whether the energy dissipation in the plasma sheath is small enough to achieve this high value of Q. Thus it is also not clear how feasible this approach for confinement laboratory plasmas with much more moderate parameters than in a fusion reactor.

We propose confining the plasma by electromagnetic pressure in a lower frequency range such that the size of the device (minor radius of torus) is much smaller than the vacuum wavelength. In this case the electromagnetic field is created by currents in the conducting shell, where no antennas and cavity modes are involved. The rf design is relatively simple. The range of confining frequencies is

$$\frac{c_s}{a} \lesssim f \ll \frac{c}{a},$$

where c_s is plasma sound speed and a is the minor radius of the torus. In practical cases $f > 100$ kHz. In this regime for fixed magnetic pressure the electric field amplitude is proportional to ω. Operation at lower frequencies reduces Ohmic loses in the conducting walls and reduces the amplitude of the rf electric field. The confining condition $\omega \ll \omega_{pe}$ is satisfied by many orders of magnitude which probably reduces plasma energy loss in this case. These factors make the proposed confinement concept more attractive.

One should note that the first ideas on rf plasma confinement in this frequency range were put forward in the late 1950s followed by some experimental studies. The ideas evolved into a "traveling-wave system" in which rf fields are produced by a set of conductors (winded in the poloidal direction) carrying high-frequency currents whose phase is shifted between adjacent conductors by a constant angle θ. In this system the amplitude of magnetic field propagates in the toroidal direction and the mean value of the square of the magnetic field increases with minor radius, see Refs. 16 and 17. This system is quite different from the confinement concept proposed here but both of them have common features such that the well documented results from the first can be used for understanding the latter.

In order to confine the plasma in our case, the polarization of the electromagnetic field at the plasma boundary must be close to circular. Indeed, the rf pressure exerted by the linearly polarized field oscillates with frequency 2ω resulting in oscillations of plasma boundary and excitations of sound waves in plasma while for circular polarization the rf pressure is time independent. This is more pronounced at lower frequencies. But probably more important is that the plasma boundary supported by the circularly polarized field is stable when it is close enough to the conducting shell (without taking into account kinetic effects) while it is unstable when supported by the linearly polarized field. The situation is analogous to electromagnetic support of liquid metals against gravity (to avoid contact with walls) routinely used in metal casting.[8] The detailed comparison of circular and linear polarizations is made in Sec. II.

A configuration in which three-dimensional plasma surface is surrounded by the electromagnetic field with nearly circular polarization is possible in toroidal geometry. The details of the proposed configuration as well as the results of numerical modeling and possible parameters of practical plasma confinement devices are presented in Sec. III.

II. CIRCULAR VERSUS LINEAR POLARIZATION

Here we compare the application of circular and linear wave polarizations for the plasma confinement. Both equilibrium and stability of plasma in the plane geometry are considered. We show that in the case of relatively low wave frequencies (the case of interest in our study) plasma can be confined by the field with circular polarization but not linear.

A. Quasilinear force of electromagnetic pressure

Consider the one-dimensional model in which uniform unmagnetized plasma occupies region $z > 0$. In the case of linear wave propagation, assuming time dependence $\propto \exp(-i\omega t)$ and local spatial dispersion, the electromagnetic field in plasma is defined by specifying tangential components of the electric field at $z=0$, $\boldsymbol{E} = E_x \boldsymbol{e_x} + E_y \boldsymbol{e_y}$, and assuming that there is no rf field at $z \to \infty$. The specification of the rf field in plasma in this way is independent on how the fields are generated in the region $z < 0$. We find the solution of wave equations with these boundary conditions for two plasma models, resistive MHD, and collisionless two fluid MHD models.

In the resistive MHD model the linearized Ohm's law defines $\boldsymbol{j} = \sigma \boldsymbol{E}$. Combining it with Maxwell's equations one finds

$$\boldsymbol{B} = \frac{ck_z}{\omega}(-E_y \boldsymbol{e_x} + E_x \boldsymbol{e_y}),$$

where $k_z = (1+i)/\lambda_s$, $\lambda_s = c/\sqrt{2\pi\sigma\omega}$-field skin depth. Because k_z has both real and imaginary parts, the phase of the rf field is a function of z. Then, for circular polarization, $E_x = E_0 \exp(-i\omega t + ik_z z)$, $E_y = iE_x$ and $\boldsymbol{E}, \boldsymbol{j}$ are at an angle of $\pi/4$ with respect to $\boldsymbol{B}$. Force exerted on the plasma per unit volume

$$\boldsymbol{f} = \frac{1}{c}\boldsymbol{j} \times \boldsymbol{B} = \frac{|\boldsymbol{B_0}|^2}{4\pi\lambda_s} \exp(-2z/\lambda_s)\boldsymbol{e_z} = -\boldsymbol{\nabla}\frac{|\boldsymbol{B}(t,z)|^2}{8\pi}, \qquad (1)$$

where

$$|\boldsymbol{B_0}| = \left|\frac{ck_z}{\omega}E_0\right|.$$

Integration over z results in total electromagnetic pressure on plasma

$$\int_0^\infty f dz = \frac{|\boldsymbol{B_0}|^2}{8\pi}.$$

The force $\boldsymbol{f}$ and the total electromagnetic pressure are time independent for the circular polarization. As will be seen in the subsequent analysis, this time independence of $\boldsymbol{f}$ remains valid also for the case of nonlinear field penetration in the plasma.

For linear polarization, $E_x = E_0 \exp(-i\omega t + ik_z z)$, $E_y = 0$ and $\boldsymbol{E}, \boldsymbol{j}$ are perpendicular to $\boldsymbol{B}$. In this case

$$\boldsymbol{f} = \frac{|\boldsymbol{B_0}|^2}{4\pi\lambda_s} \exp(-2z/\lambda_s)\frac{1}{2}\left[1 + \sqrt{2}\sin\left(2\omega t - \frac{2z}{\lambda_s} + \frac{\pi}{4}\right)\right]\boldsymbol{e_z} = -\boldsymbol{\nabla}\frac{|\boldsymbol{B}(t,z)|^2}{8\pi},$$

$$\int_0^\infty f dz = \frac{|\boldsymbol{B_0}|^2}{8\pi}\frac{1}{2}(1 + \sin 2\omega t).$$

The electromagnetic pressure is time dependent for linear polarization, ranging from 0 to $|\boldsymbol{B_0}|^2/8\pi$. Thus, when the rf period is comparable to plasma thermal expansion time (without electromagnetic pressure) then the plasma boundary moves substantially during the part of the cycle when electromagnetic pressure is smaller than plasma pressure. Due to this plasma cannot be confined by linearly polarized electromagnetic field with the frequency comparable or smaller than the inverse plasma thermal time. In the case of elliptic polarization with $E_y = i\alpha E_x$,

$$\int_0^\infty f dz = \frac{|\boldsymbol{B_0}|^2}{8\pi}\frac{1}{2}[1+\alpha^2+(1-\alpha^2)\sin 2\omega t].$$

Thus, the closer the polarization is to circular, the better the plasma confinement.

In the collisionless two fluid model, linear plasma response is defined by the isotropic dielectric tensor. Such that in the case of interest, $\omega \ll \omega_{pe}$, plasma conductivity is

$$\sigma = \frac{i}{4\pi}\frac{\omega_{pe}^2}{\omega}.$$

In this case $k_z = i/\lambda_s$, with $\lambda_s = c/\omega_{pe}$. There is no phase shift along the z axis. For circular polarization $\boldsymbol{E}$ is parallel to $\boldsymbol{B}$ while $\boldsymbol{j}$ is perpendicular to $\boldsymbol{B}$. Force per unit volume

$$\boldsymbol{f} = \frac{|\boldsymbol{B_0}|^2}{4\pi\lambda_s}\exp(-2z/\lambda_s)\boldsymbol{e}_z = -\nabla\frac{|\boldsymbol{B}(t,z)|^2}{8\pi}$$

is defined by the same formula as in the resistive MHD model with the only difference in the definition of λ_s. For linear polarization $\boldsymbol{E}$ and $\boldsymbol{j}$ are perpendicular to $\boldsymbol{B}$,

$$\boldsymbol{f} = \frac{|\boldsymbol{B_0}|^2}{4\pi\lambda_s}\exp(-2z/\lambda_s)\frac{1}{2}(1-\cos 2\omega t)\boldsymbol{e}_z = -\nabla\frac{|\boldsymbol{B}(t,z)|^2}{8\pi},$$

$$\int_0^\infty f dz = \frac{|\boldsymbol{B_0}|^2}{8\pi}\frac{1}{2}(1-\cos 2\omega t).$$

In the above results f is calculated as $\boldsymbol{f} = \boldsymbol{j}\times\boldsymbol{B}/c$, where all other two fluid contributions to the force[18] are zero in this geometry. The results for f in the two fluid model relate to the ponderomotive force on single particle in rf field[19] $\boldsymbol{f} = -n\nabla\Phi$, where $\Phi = e^2|\boldsymbol{E}|^2/2m_e\omega^2$ for circular polarization and $\Phi = e^2|\boldsymbol{E}|^2/4m_e\omega^2$ for linear polarization. Here $\boldsymbol{E}$ is the electric field amplitude.

Thus, in spite of the difference in the relative directions of field components in resistive MHD and two fluid models, the results for the electromagnetic pressure are similar. Therefore the conclusions about the applicability of circular and linear polarizations for plasma confinement in this frequency range are the same for the two models. These results are expected to be valid in the general case of a hot dense plasma (for which fluid approximation $\omega/k_z > v_{Te}$ is not satisfied) and nonlinear penetration of rf field as long as the condition $\omega \ll \omega_{pe}$ is satisfied. Results in Ref. 20 show that in the hot plasma the skin depth of the circularly polarized wave is a few c/ω_{pe} which is also the scale length of the plasma density profile when plasma is supported by electromagnetic pressure. In this reference the approximation that the electrons are cold in the direction transverse to the rf wave vector is made. Additional first-principles calculations are needed for a more definite conclusion about the applicability of the results in the hot plasma case.

B. One-dimensional equilibrium

Consider plasma in volume $0 \le z \le a$, uniform in x and y. At $z=0$ tangential components of the rf electric field are specified and plasma is bounded by a conducting wall at $z=a$. Plasma motion in the applied rf field is modeled by resistive MHD equations,

$$\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B} = \frac{1}{\sigma}\boldsymbol{j},$$

$$\rho\frac{d\boldsymbol{v}}{dt} = -\nabla p + \frac{1}{c}\boldsymbol{j}\times\boldsymbol{B},$$

where σ is a constant (does not depend on plasma density). For the case of circular polarization there is a self-consistent equilibrium solution in which $\boldsymbol{v}=0$. In this case electromagnetic force per unit volume is defined by Eq. (1), it is time independent and it is balanced by plasma pressure gradient,

$$\nabla p + \nabla\frac{|\boldsymbol{B}(z)|^2}{8\pi} = 0.$$

Assuming for simplicity that plasma compression is isothermal, $p/\rho=$const, and taking into account that $\lambda_s \ll a$ we find

$$p = p_0 - \frac{|\boldsymbol{B_0}|^2}{8\pi}\left(e^{-(2z/\lambda_s)} - \frac{\lambda_s}{2a}\right), \quad (2)$$

where p_0 is the plasma pressure before the rf field is applied. Equation (2) shows that in this model plasma is supported by electromagnetic pressure and the scale length of the plasma density gradient at the boundary is $\lambda_s/2$. At $z=0$, $p \approx p_0 - |\boldsymbol{B_0}|^2/8\pi$ and $p<0$ if $|\boldsymbol{B_0}|^2/8\pi > p_0$. Plasma is completely expelled from the boundary region if magnetic pressure of the rf field exceeds the initial plasma pressure. Thus, this condition is necessary for plasma confinement. Depletion of plasma density to zero near the boundary creates difficulty in modeling plasma confinement since the resistive MHD model becomes invalid there. This difficulty is hard to remove even by increasing plasma resistivity near the boundary which reduces the gradient of electromagnetic force and makes the plasma pressure profile less steep.

In the considered case the interaction of plasma and electromagnetic field is nonlinear but the force f is time independent. This is the property of the circularly polarized field. It remains valid also for arbitrary dependence of σ on plasma density, for plasma equilibrium modeled by collisionless two fluid MHD equations, and for the general case of hot plasma.[20] Because the latter was studied with some approximations, additional first-principles modeling is required for more definite results in the hot plasma case.

There is no equilibrium solution with $\boldsymbol{v}=0$ for the linearly polarized field. In this case v_z oscillates on the harmonics of the driving frequency. One can achieve a dynamic equilibrium of plasma in this case if the applied frequency is so high that plasma expands a little during the part of the field period when electromagnetic pressure is smaller than plasma pressure. Our study, however, concentrates on lower frequencies for which nearly circular field polarization is the only feasible option for plasma confinement.

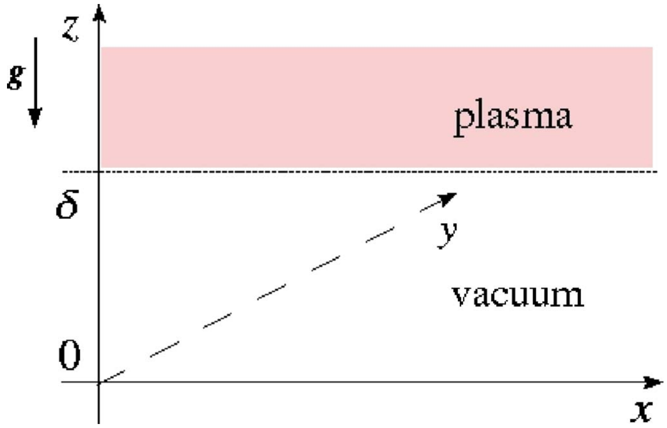

FIG. 1. (Color online) Geometry for stability analysis.

C. Stability analysis in plane geometry

We carry out a simplified stability analysis of plasma boundary for different polarizations of applied electromagnetic field. Consider plasma as a perfectly conducting ($\lambda_s \ll a$) incompressible fluid in the gravitational field, occupying region $z > \delta$, see Fig. 1. This model corresponds to an ideal MHD plasma model (except for the incompressibility) since the oscillating magnetic field does not penetrate into the plasma from the outside (frozen-in condition). Electromagnetic field is applied in plane $z=0$. There is a vacuum gap in the region $0 < z < \delta$. We assume that the applied field frequency ω is large enough such that the displacement of plasma boundary on a time scale $1/\omega$ can be neglected. In equilibrium plasma is supported by time averaged electromagnetic pressure p_{em}.

In this simple geometry $\boldsymbol{g}$ plays the role of instability drive. In realistic toroidal geometry with curved instantaneous confining magnetic field there is a possibility of current or pressure driven instabilities (detailed study of which is the subject of future analysis). The results for the restoring force acting on the perturbed plasma boundary calculated here should also be valid in realistic geometry. Thus the conclusions made here about plasma stability for different field polarizations are expected to hold in the realistic case.

We consider three different cases corresponding to different applied electromagnetic fields at $z=0$. 1. Circular polarization, $\boldsymbol{E}=E_0(\boldsymbol{e_x}+i\boldsymbol{e_y})$. 2. Linear polarization, $\boldsymbol{E}=E_0\boldsymbol{e_x}$. These two cases correspond to specifying voltages applied to conducting shell of a vessel containing plasma. 3. Circular polarization, $\boldsymbol{B}=B_0(\boldsymbol{e_x}+i\boldsymbol{e_y})$. In this case currents in the shell are specified. We perform stability analysis similar to one given in Chap. 5 in Ref. 21 for plasma boundary supported by magnetic pressure. Let $z_s-\delta=A\cos(-\Omega t+k_x x+k_y y)$ is the displacement of the plasma surface. We make reasonable assumptions that $\Omega \ll \omega$, $\omega\delta/c \ll 1$, and $\omega/c \ll k_0=\sqrt{k_x^2+k_y^2}$.

Plasma motion satisfies

$$\rho_0 \frac{d\boldsymbol{v}}{dt} = -\boldsymbol{\nabla} p + \rho_0 \boldsymbol{g}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{v}=0.$$

In equilibrium, $p=p_0-\rho_0 g(z-\delta)$. Pressure balance on the perturbed plasma surface is

$$p_0 - \left(\rho_0 g + \frac{\rho_0\Omega^2}{k_0}\right) A\cos(-\Omega t + k_x x + k_y y) = p_{\mathrm{em}}, \qquad (3)$$

where p_{em} is electromagnetic pressure at the location of the perturbed plasma-vacuum boundary. Maxwell's equations are solved to find electromagnetic fields in the vacuum region which are subject to boundary conditions at $z=0$ and $z=z_s$. Then, after straightforward but tedious calculations we find p_{em} for the three cases.

In the first case,

$$p_{\mathrm{em}} = \frac{B_0^2}{8\pi}\left[1 - k_0 \frac{e^{-k_0\delta}+e^{k_0\delta}}{e^{k_0\delta}-e^{-k_0\delta}} A\cos(-\Omega t + k_x x + k_y y)\right],$$

where $B_0=cE_0/\omega\delta$ is the amplitude of equilibrium magnetic field in vacuum. From this equation one can see that the correction to electromagnetic pressure due to plasma boundary perturbation is stabilizing, it pushes plasma boundary back to equilibrium. A combination of this result with Eq. (3) gives

$$\Omega^2 = \frac{k_0}{\rho_0}\left(p_0 k_0 \frac{e^{-k_0\delta}+e^{k_0\delta}}{e^{k_0\delta}-e^{-k_0\delta}} - \rho_0 g\right).$$

Thus, since

$$k_0\delta \frac{e^{-k_0\delta}+e^{k_0\delta}}{e^{k_0\delta}-e^{-k_0\delta}} \geq 1,$$

the equilibrium is stable if $\rho_0 g\delta/p_0 \leq 1$. Therefore in the case of circular polarization and fixed applied voltages plasma equilibrium is stable if the vacuum gap is small enough. A similar conclusion is expected in cylindrical geometry from physics considerations.

In the second case,

$$p_{\mathrm{em}} = \frac{B_0^2}{16\pi}\left[1 - 2\frac{k_y^2}{k_0} \frac{e^{-k_0\delta}+e^{k_0\delta}}{e^{k_0\delta}-e^{-k_0\delta}} A\cos(-\Omega t + k_x x + k_y y)\right].$$

The pressure correction is stabilizing only if $k_y \neq 0$ and the equilibrium is unstable if $k_y=0$ and $k_x \neq 0$. This result shows that linear polarization is not suitable for plasma confinement from the stability argument.

In the third case,

$$p_{\mathrm{em}} = \frac{B_0^2}{8\pi}\left[1 - k_0 \frac{e^{k_0\delta}-e^{-k_0\delta}}{e^{k_0\delta}+e^{-k_0\delta}} A\cos(-\Omega t + k_x x + k_y y)\right].$$

This result is different from the first case, and if $k_0\delta \ll 1$ (the range of interest) the correction to electromagnetic pressure is negligible, meaning that the equilibrium is unstable. Thus, for plasma confinement it is important to fix applied voltages rather than currents. This situation is similar to a capacitor filled with dielectric. The equilibrium of dielectric is stable or unstable depending on whether one fixes charge or voltage on the capacitor.

Stability analysis in this section is carried out for the fluid plasma model. It should also be valid (with some modifications) in hot plasma since the boundary condition on tangential components of the electric field ($E_t \sim 0$) at the perturbed plasma boundary (which define perturbation of magnetic pressure) should be satisfied in the hot plasma as well.

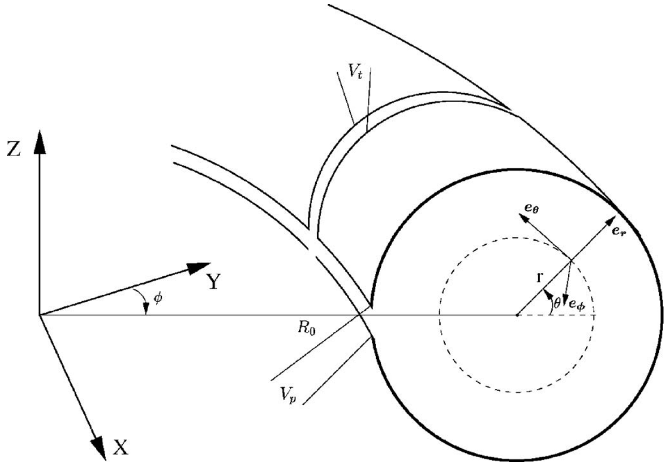

FIG. 2. Geometry for plasma confinement.

III. PLASMA CONFINEMENT IN TOROIDAL GEOMETRY

A. Plasma confinement scheme

Analysis in the previous section shows that circular polarization of the electromagnetic field is required for plasma confinement from both equilibrium and stability considerations. It is probably impossible to surround the three-dimensional plasma boundary by an exactly circularly polarized electromagnetic field. However one can surround the 3D boundary by the electromagnetic field with nearly circular polarization. The proposed plasma confinement scheme in toroidal geometry is presented in Fig. 2. The conducting toroidal shell, with minor and major radii a and R_0, has two (or more) insulated cuts. One of them is in the horizontal direction-toroidal gap, and another is in the vertical direction-poloidal gap. The width of the gaps is assumed to be much smaller than the poloidal circumference of the shell and smaller than the width of the vacuum layer between the shell and the plasma boundary. RF voltages with frequency ω and amplitudes V_p and V_t are applied, respectively, to toroidal and poloidal gaps such that the condition

$$\frac{V_p}{2\pi a} = \frac{V_t}{2\pi R_0} \tag{4}$$

is approximately satisfied. The phase shift between V_p and V_t is $\pi/2$. For practical purposes, $f \gtrsim 100$ kHz and V_p is starting from around 100 V. Because the voltages can be quite high, one can make several poloidal and toroidal gaps in the shell in order to reduce the voltage drop per gap. Field frequency is bounded from the above by the requirement that the wavelength in vacuum is much larger than the size of the machine,

$$\frac{c}{f} \gg a. \tag{5}$$

In our analysis we adopt an ideal MHD model for plasma and assume that there is a vacuum layer between the plasma boundary and the conducting shell. One can also adopt resistive MHD model assuming that highly resistive boundary plasma extends to the shell's surface. In the latter case, however, there is difficulty related to the edge plasma density depletion. As discussed in the previous section, edge plasma is pushed inward by the electromagnetic pressure even if the edge plasma resistivity is high. Our limited analysis based on the resistive MHD model indicated that the conclusions based on this model are similar to the results presented here, based on an ideal plasma model. Because initially plasma is unmagnetized, the oscillating field does not penetrate in the plasma (frozen-in condition). Thus the plasma in our model is an ideally conducting fluid. In the realistic case of resistive (or collisionless two fluid plasma) oscillating field penetrates in the skin layer which is much smaller than a for the considered range of parameters.

If only voltage V_p is applied to the toroidal gap, it drives the poloidal current in the shell. The opposite image current is driven on the plasma surface. These currents generate the oscillating toroidal magnetic field B_ϕ in the vacuum layer which in the limit of Eq. (5) satisfies

$$B_\phi = B_0 \frac{R_0}{R}, \tag{6}$$

where R is the coordinate along the major radius of the torus. Thus, the electromagnetic pressure on the plasma on the inboard side ($\theta = \pi$) is larger than the one on the outboard side ($\theta = 0$). The resulting force pushes plasma in the outboard direction which is similar to the case of toroidal Θ-pinch when only the toroidal equilibrium magnetic field is present.

In our analysis we assume that plasma confinement in the applied fields is toroidally symmetric. This is a reasonable approximation because of the condition of Eq. (5) and assuming that the width of the poloidal gap is smaller than the width of the vacuum layer. Electromagnetic field in the vacuum layer obeys Maxwell's equations in toroidal coordinates. We average these equations over coordinate ϕ and consider equations for the averaged components. We find

$$B_\theta = -\frac{c}{i\omega}\left(\frac{\cos\theta}{R_0 + r\cos\theta}E_\phi + \frac{\partial E_\phi}{\partial r}\right), \tag{7}$$

$$B_\phi = \frac{c}{i\omega}\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial \theta}\right]. \tag{8}$$

At the shell's boundary $r = a$ the averaged field is

$$E_\phi = \frac{V_t}{2\pi R}. \tag{9}$$

Equation (9) defines one of the boundary conditions at $r = a$, the condition for the averaged E_ϕ component. One should note that the E_ϕ itself is not toroidally symmetric (it is nonzero at the gap and zero on the surface of the shell). Because of the condition of Eq. (5), however, currents in the shell and magnetic field are symmetric (with small deviation in the vicinity of the gap). Thus, the averaged equations with this boundary condition are appropriate for finding the magnetic field in the vacuum layer. Since magnetic pressure is the dominant part of the electromagnetic pressure on the plasma surface, the use of the above equations is justified for studying plasma confinement.

Voltage V_t drives the toroidal current in the shell. Oscillating magnetic flux through the area $R < R_0 - a$ induces the toroidal electric field. This electric field drives current on the plasma surface in the direction opposite to the current in the shell. This mechanism of current drive in plasma is similar to inductive current drive in tokamaks where the variable magnetic flux is directed through the central part of the torus.

For the cases of interest the term $\partial E_\phi / \partial r$ dominates in Eq. (7). Thus, locally, $B_\theta \sim E_\phi(r=a)/d$, where $d(\theta)$ is the local width of the vacuum layer. In this case the electromagnetic pressure depends on the distance from plasma surface to the shell, the closer plasma to the shell the stronger electromagnetic pressure. Physically when d gets smaller, local surface impedance of the shell is reduced and for fixed voltage the current in the shell increases by increasing the local electromagnetic pressure (note importance of fixed voltage). The case when toroidal voltage V_t is applied is completely different from the case with poloidal voltage V_p, in the latter the electromagnetic pressure does not depend on d locally (in the limit of cylinder rf pressure does not depend on θ for arbitrary plasma shape). Therefore when the two voltages are applied simultaneously the net time averaged outward force due to V_p is balanced by the inward force due to V_t when plasma is shifted outward, such that $d(\theta=0) < d(\theta=\pi)$. The situation is analogous to tokamak where toroidal current (B_θ) is necessary for the plasma equilibrium. The outward shift in our case is analogous to the Shafranov shift in tokamak.

In the case of applied V_t one should note that because on the plasma surface the tangential electric field is nearly zero (it is small for moving plasma surface) the magnetic flux in the region $R < R_0 - a$ is compensated by the magnetic flux in the vacuum layer. Since the width of the layer is much smaller than $R_0 - a$, the magnetic flux in the region $R < R_0 - a$ should be much smaller than in the case without plasma (current in the shell is compensated by mirror current on the plasma surface). In other words surface impedance on the inside part of the shell is much smaller than the one on the outside and current flows on the inner surface of the shell. Because of this, the magnetic energy outside of the torus should be relatively small.

The condition of Eq. (4) insures that in the limit $a/R_0 \to 0$ and $d \ll a$ the electromagnetic field at the plasma boundary is circularly polarized. Let $B_{\theta 1}$, $B_{\phi 1}$ and $B_{\theta 2}$, $B_{\phi 2}$ are magnetic field amplitudes at the plasma boundary at points $\theta=0$ and $\theta=\pi$, respectively. If $a/R_0 \ll 1$, then in equilibrium, $B_{\theta 1} \approx B_{\phi_2}$. Using Eq. (6) we get

$$\frac{B_{\phi 1}}{B_{\theta 1}} \approx 1 - 2\frac{a}{R_0}. \tag{10}$$

This estimate shows how close the field polarization is to the circular one in the torus with finite aspect ratio. Electromagnetic pressure at the plasma boundary oscillates with frequency 2ω about its average value with amplitude $\sim 2a/R_0$. These pressure oscillations drive sound waves in plasma with similar amplitude. Thus, the larger the aspect ratio of the torus, the more quiescent the plasma equilibrium is expected.

The plasma confinement scheme presented here is analogous to suggested confinement of liquid metal inside a toroidal inductor.[22] In this reference equilibrium of liquid metal in the gravitational field and oscillating magnetic fields is calculated analytically in the limit of a large aspect ratio.

B. Numerical modeling of toroidal confinement

We consider the 2D toroidally symmetric geometry. At $t=0$ plasma density and pressure are ρ_0 and p_0. We define sound speed as $c_s = \sqrt{p_0/\rho_0}$ and use the following normalization, $\rho = \rho_0 \tilde{\rho}$, $p = p_0 \tilde{p}$, $v = c_s \tilde{v}$, $r = a\tilde{r}$, $t = (a/c_s)\tilde{t}$, $\omega = (c_s/a)\tilde{\omega}$, $B = \sqrt{4\pi p_0}\tilde{B}$, $E = \sqrt{4\pi p_0}(c_s/c)\tilde{E}$. We consider equations for dimensionless quantities and omit the $\tilde{}$ symbol.

In the vacuum layer the electromagnetic field satisfies Maxwell's equations

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \nabla \times \boldsymbol{B} = \left(\frac{c_s}{c}\right)^2 \frac{\partial \boldsymbol{E}}{\partial t} \tag{11}$$

which are transformed to toroidal coordinates and averaged over ϕ. Plasma is a superconducting fluid,

$$\frac{\partial \boldsymbol{v}}{\partial t} = -(\boldsymbol{v}\nabla)\boldsymbol{v} - \frac{\gamma}{\rho^{2-\gamma}}\nabla\rho + \eta \triangle \boldsymbol{v}, \tag{12}$$

$$\frac{\partial \rho}{\partial t} = -\nabla(\rho \boldsymbol{v}), \tag{13}$$

where $\gamma = 5/3$, η is dimensionless viscosity required for numerical stability. The motion of plasma boundary $r_p(t,\theta)$ satisfies

$$\frac{\partial r_p}{\partial t} = v_r - \frac{\partial r_p}{\partial \theta}\frac{v_\theta}{r_p}.$$

The boundary conditions at the plasma-vacuum boundary are (see Chap. 5 in Ref. 21)

$$\boldsymbol{n} \times \boldsymbol{E} = u\boldsymbol{B}, \quad u = \boldsymbol{n}\boldsymbol{v}, \tag{14}$$

$$p = \frac{1}{2}\boldsymbol{B}^2 - \frac{1}{2}\left(\frac{c_s}{c}\right)^2 \boldsymbol{E}^2 + p_f(t,r). \tag{15}$$

In the above equations $\boldsymbol{n}$ is the normal vector to the plasma boundary. Equation (15) is the pressure balance at the plasma boundary. The term $p_f(t,r)$ is for supporting plasma boundary when the electromagnetic field is not completely turned on. $p_f(t,r)$ reduces to zero at the same rate with which the electromagnetic field is turned on. The negative contribution to the rf pressure from the electric field is due to the force acting on surface charges in the electric field normal to the plasma boundary.

The boundary conditions at $r=a$,

$$E_\phi = \frac{R_0}{R_0 + \cos\theta}[1 - e^{-(t/\triangle t)^2}]E_0 \sin \omega t, \tag{16}$$

$$E_\theta = \frac{2\sqrt{\pi}}{\triangle \theta} e^{-(\theta-\pi/\triangle\theta)^2}[1 - e^{-(t/\triangle t)^2}]E_0 \cos \omega t. \tag{17}$$

In the above equations dimensional E_0 satisfies $E_0 = V_t/2\pi R_0 = V_p/2\pi a$. Since $R = R_0 + \cos\theta$, the boundary condition for E_ϕ is in line with Eq. (9). At $t=0$ electric and

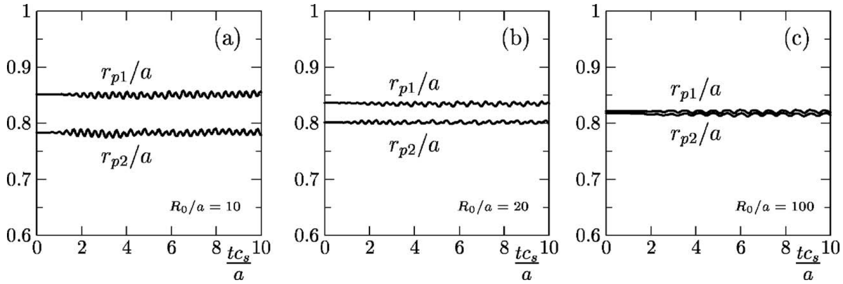

FIG. 3. Plasma radius vs time. r_{p1} at $\theta=0$, r_{p2} at $\theta=\pi$. (a) $R_0/a=10$; (b) $R_0/a=20$; (c) $R_0/a=100$.

magnetic fields in the vacuum layer are zero. The turning on time $\triangle t$ should be long enough, such that $\triangle t\omega \gg 1$. If this condition is not satisfied a steady state component of magnetic field is excited (along with the oscillating part). Equation (17) is the boundary condition for the E_θ component. The field is concentrated at the gap and it is zero away from the gap. The width of the gap is $\triangle\theta$ and it is located on the inboard side of the torus. By integrating Eq. (17) over θ one finds that the total poloidal loop voltage amplitude is V_p.

Our results are not sensitive to the value of $\triangle\theta$ in Eq. (17) as long as the contribution from the electric field in Eq. (15) is negligible. The main electric field contribution in pressure balance comes from the E_r component which is largest in the vicinity of the gap. Because $E \propto \omega$ the electric field contribution becomes noticeable at higher frequencies. For the studied parameters this contribution is no more than a few percent of magnetic pressure. Also in a more realistic case the width of the gap is smaller than the width of the vacuum layer d (this case is not considered here because of limitations in computing power) and E_r drops significantly on the distance d (dipole field) reducing the effect of the gap geometry on the local pressure profile. Due to these reasons there is probably no need for the accurate modeling of the gap in the plasma equilibrium study (as long as the electric field contribution to the rf pressure is negligible), such that one can use an arbitrary $E_\theta(\theta)$ profile at $r=a$ as long as its integral corresponds to a proper loop voltage. In this case the proper B_ϕ profile of Eq. (6) is achieved by adjusting the $\partial E_r/\partial\theta$ term in Eq. (8).

To study the equilibrium plasma properties we solve Eqs. (11)–(13) as an initial value problem. We use an explicit predictor-corrector time advancement algorithm. Transition to coordinates which are tailored to the geometry of the plasma surface and toroidal shell is made. The small coefficient c_s^2/c^2 in Eq. (11) is very restrictive in the explicit time advancement algorithm. Since the result does not depend on this coefficient as long as the condition of Eq. (5) is satisfied, we make it larger [reduce c such that Eq. (5) is still valid] in Eq. (11) only in order to make simulations more efficient.

Simulations are performed in two stages. Initially, plasma has a circular cross section of radius $r_p/a=0.95$ and is supported by $p_f(r)$. In the first stage, the boundary electric field of Eqs. (16) and (17) is gradually turned on and $p_f(t,r)$ is turned off at the same rate. During this stage plasma volume experiences significant transformation which excites oscillations of the plasma bulk. These oscillations have relatively large amplitude and decay on a time scale larger than the computationally feasible time interval. Due to this reason we start simulations again (second stage) with the shape of the plasma boundary taken from the first stage such that it is close to the time-averaged equilibrium plasma profile. We assume that at the starting moment of the second stage, $t=0$, density is uniform within the plasma boundary and again plasma is supported by suitable $p_f(r,\theta)$ which is uniform along the initial plasma boundary. The same boundary electric field is gradually turned on while $p_f(t,r,\theta)$ is reduced to zero. When the boundary electric field amplitudes reach steady state oscillations plasma is close to dynamic equilibrium. At this time plasma is fully supported by electromagnetic pressure.

We present the results of time evolution of the electromagnetic field and plasma during the second stage. Our calculations are carried out for the following values of dimensionless parameters, $\omega=10$, $\triangle t=1.5$, $c_s/c=10^{-3}$, $\triangle\theta=0.25$, $E_0=3$, $\eta=10^{-2}$, total time of calculation is $t=10$. We present the results for three aspect ratios, $R_0=10$, $R_0=20$, $R_0=100$.

Plasma radii r_{p1} and r_{p2} corresponding to poloidal coordinates $\theta_1=0$ and $\theta_2=\pi$ versus time are shown in Figs. 3(a)–3(c). Since $r_{p1}>r_{p2}$ plasma is shifted outboard. This shift is larger for smaller R_0/a. Plasma boundary oscillates with frequency 2ω corresponding to the frequency of oscillation of electromagnetic pressure due to some ellipticity in the field polarization. Using linearized Eqs. (12) and (13) one

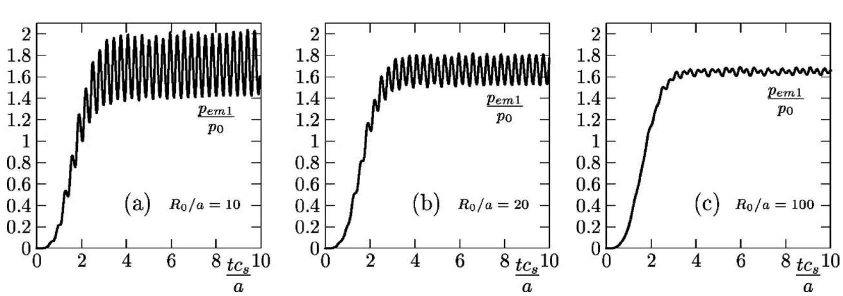

FIG. 4. Electromagnetic pressure vs time at $\theta=0$. (a) $R_0/a=10$; (b) $R_0/a=20$; (c) $R_0/a=100$.

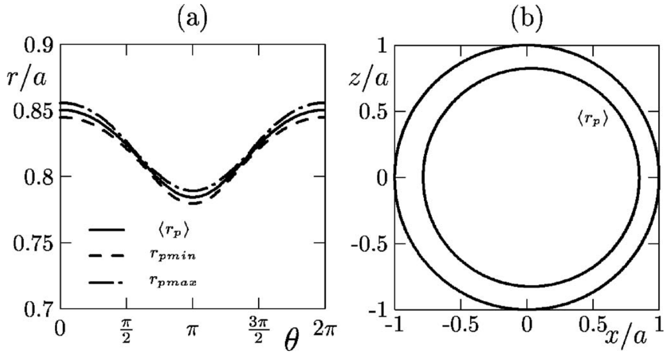

FIG. 5. (a) $\langle r_p \rangle$, $r_{p\min}$, $r_{p\max}$ vs θ; (b) $\langle r_p \rangle$ in poloidal cross section. Time averaging is in the vicinity of $tc_s/a=8$, $R_0/a=10$.

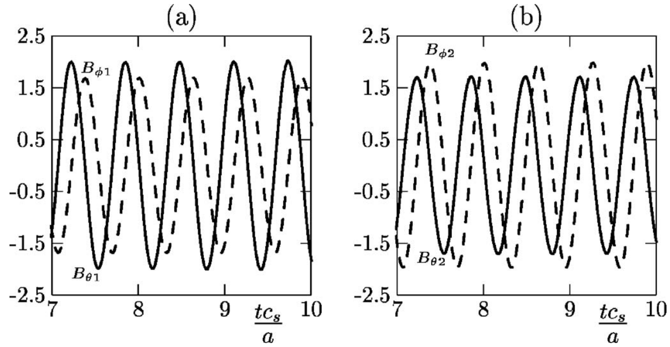

FIG. 6. Magnetic field components at plasma boundary vs time. (a) B_θ, B_ϕ at $\theta=0$; (b) B_θ, B_ϕ at $\theta=\pi$. In both cases $R_0/a=10$.

can estimate the amplitude of oscillations of r_p. In dimensionless units $\triangle r_p \sim \triangle p/\sqrt{\gamma}\omega$. $\triangle p$ is equal to the amplitude of oscillation of magnetic pressure, $\triangle p \sim 2a/R_0$. This estimate gives $\triangle r_p/a \sim 1\%$ for $R_0/a=10$ which is in line with the result in Fig. 3(a). One should note that in spite of the fact that $\triangle r_p/a$ is quite small at this frequency, the amplitude of oscillations of plasma pressure and density is still relatively large, $\triangle p/p_0 \sim 2a/R_0=20\%$ for $R_0/a=10$. Excitation of sound waves with such amplitude is observed in simulations. In kinetic theory these oscillations are strongly damped which could provide plasma heating mechanism in the proposed confinement concept.

Plots of electromagnetic pressure versus time at the location on the plasma surface corresponding to $\theta_1=0$ are presented in Figs. 4(a)–4(c). p_{em} is defined by Eq. (15) without the term p_f. When electromagnetic fields reached steady state, the pressure oscillates with frequency 2ω about its average value. This averaged value is close to $p_{\mathrm{em}}/p_0 \sim 1.7$ because plasma is compressed from the initial state with $r_p/a=0.95$, $p/p_0=1$. The amplitude of electromagnetic pressure oscillations at the plasma surface approximately satisfies the mentioned scaling $\triangle p_{\mathrm{em}}/\langle p_{\mathrm{em}}\rangle \sim 2a/R_0$.

Figure 5(a) shows time averaged plasma radius $\langle r_p \rangle$, maximum and minimum plasma radii versus poloidal angle θ for $R_0/a=10$. The averaging and calculation of maximum and minimum plasma radius is done on a time interval of one field period in the vicinity of $tc_s/a=8$. The deviation of the plasma surface from its time averaged position is of the order of 1% of the minor radius. Plasma surface is almost stationary near locations $\theta=\pi/2$ and $\theta=3\pi/2$ meaning that the field polarization is almost circular there. Figure 5(b) shows the location of the time averaged plasma surface in poloidal cross section. Plasma is shifted in the outboard direction in this equilibrium.

Figure 6 shows time dependence of magnetic field components $B_{\theta 1}$, $B_{\phi 1}$ ($\theta=0$) and $B_{\theta 2}$, $B_{\phi 2}$ ($\theta=\pi$) at the plasma boundary for $R_0/a=10$. They oscillate with frequency ω and the phase shift between B_θ and B_ϕ is $\pi/2$. The previously used relation $B_{\theta 1} \approx B_{\phi 2}$ is confirmed by the results in this figure.

The profile of E_r along the plasma boundary at the moment of time $tc_s/a=8.17$ (in which E_r is maximal) is shown in Fig. 7. E_r strongly varies near the gap ($\theta=\pi$) and has approximately linear θ dependence away from the gap. As the width of the gap $\triangle\theta$ gets smaller this linear dependence extends to the location of the gap $\theta=\pi$ from both sides. This is true for the examined cases in which the width of the gap is larger than the width of the vacuum layer. It is expected that the maximum value of E_r at the plasma surface near the gap would remain approximately the same as in this plot as $\triangle\theta \to 0$. The linear θ dependence of E_r can be explained by comparing the θ component of the Poynting flux $\sim E_r B_\phi$ with magnetic energy (due to B_ϕ) stored in an angular segment of the vacuum layer. Because B_ϕ is approximately uniform, the stored energy is linear with the angular size of the segment resulting in linear dependence of E_r vs θ away from the gap.

For the considered case the maximum electric field contribution to the electromagnetic pressure of Eq. (15) is much smaller than the magnetic pressure. For higher ω electric field contribution to the pressure balance can be larger (but still a small fraction of magnetic pressure for the cases of interest). Because the electric field contribution is negative the inboard location of the toroidal gap is preferable, such

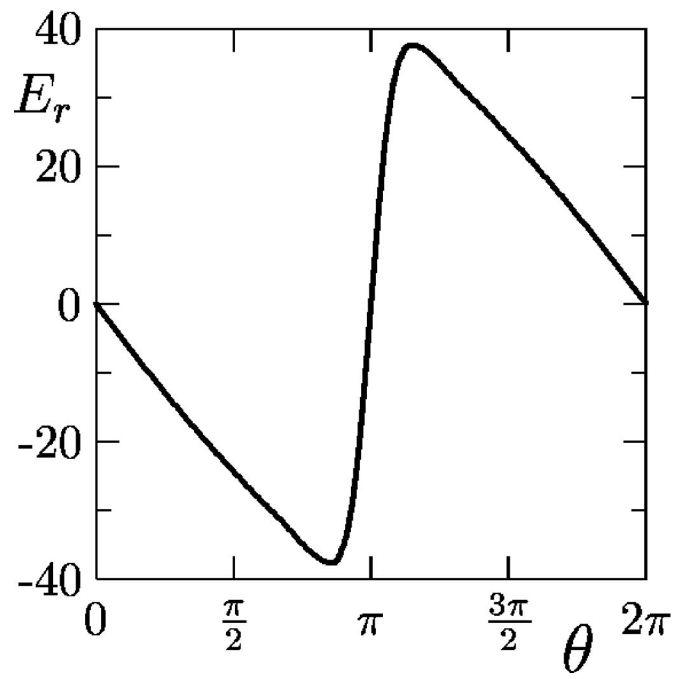

FIG. 7. E_r vs θ at plasma boundary. $tc_s/a=8.17$, $R_0/a=10$.

that the magnetic pressure imbalance due to the finite aspect ratio is partially compensated by the electric field contribution to the pressure balance near the gap.

In the presented results the field frequency $\omega a/c_s=10$ is chosen with some margin to the higher side. In this case no instability is observed in the studied (2D) model. When $\omega a/c_s\sim 1$ the field frequency is close to the natural frequency of plasma oscillations near the equilibrium. In this case plasma kinetic energy can grow in the applied electromagnetic field. Such unstable equilibria were observed for some parameters. Stable equilibria could be possible for frequencies $\omega a/c_s \ll 1$. Their study is beyond the scope of this paper.

C. Practical implementation

We estimate parameters of possible plasma confinement machines using results of the previous subsection. We consider the same dimensionless parameters, $\omega a/c_s=10$, $B/\sqrt{4\pi p_0}=2$, $E_0 c/c_s\sqrt{4\pi p_0}=3$ and take $R_0/a=20$. We estimate only the gap voltage V_p since the number of gaps in the toroidal direction can be made large enough to reduce the voltage per gap requirement. Consider compact configuration with hydrogen plasma with $R_0=100$ cm, $a=5$ cm, $n_e=10^{12}$ cm^{-3}, $T=10$ eV. For this case we find $B=40$ G, field frequency $f=1.4\cdot 10^6$ Hz, $V_p=85$ V. Power dissipated per unit area in a skin layer of copper shell for circular polarization is

$$P_s=\sqrt{\frac{\omega c^2}{2\pi\sigma}}\frac{B^2}{8\pi}.$$

Using copper resistivity $\rho=1.7\cdot 10^{-8}\ \Omega$ m we find $P_s\sim 3$ kW/m^2. The total power dissipated on the shell's surface is $P\sim 6$ kW. The device with these parameters is feasible. Configurations with lower plasma densities than that considered in this example might also be of practical interest.

An important question that one should consider is the power absorption in the plasma skin layer. For the parameters of interest the condition $\omega/k_s < v_{Te}$ (with $k_s=2\pi/\lambda_s$) is satisfied meaning that the transverse electromagnetic field (relative to the wave number k_s) in the skin layer can strongly heat plasma electrons by the collisionless mechanism. On the other hand the electrons in the skin layer are magnetized. For all cases considered here the electron Larmor radius is smaller than the skin layer depth. An electron drawing energy from the electric field in the skin layer will drift in the magnetic field and remain in the region of sufficiently strong fields until, as a result of collisions with other particles, it escapes into the low-field region. Thus the estimate of power absorption by the collisional mechanism is justified, but a more advanced first-principles modeling is required for more reliable estimates of the heating rate.

Results in Ref. 17 show that the power absorption in the skin layer can overheat plasma in the device (assuming that the absorbed power heats the whole plasma volume). In this case plasma pressure can exceed the confining magnetic pressure resulting in the loss of confinement. The situation is opposite to magnetic confinement systems in which plasma heating requires significant efforts. Assuming collisional damping in the skin layer one can estimate time τ_{ov} during which plasma is heated to thermal energy per unit volume equal to the confining magnetic pressure. In our estimates we use the result of Eq. (2.18a) in Ref. 17,

$$\tau_{ov}=\frac{5\cdot 10^{-2}T^{3/2}ak}{\sqrt{n}}, \tag{18}$$

where τ_{ov} is in s, T is plasma temperature in eV, a is minor radius in cm, k is the dimensionless number of the order of a few units, n is the plasma density in cm^{-3}. The power consumed on plasma heating is

$$P_h=\frac{B^2}{8\pi\tau_{ov}}V,$$

where V is the plasma volume. In the cases when τ_{ov} is smaller than the energy confinement time plasma can become overheated. If this is the case, then in order to maintain plasma one should reduce the energy confinement time by placing a limiter or some kind of divertor (with a local change of confining field topology) to limit plasma radius (and pressure). For the configuration considered above $\tau_{ov}\sim 24\cdot 10^{-6}$ s, $P_h\sim 8$ kW. In these estimates we used $r_p/a=0.8$ and $k=3$. The plasma heating power is comparable to the power loss in the shell making it a sizable contribution to the power balance. For higher temperature plasmas relative contribution of P_h to the power balance is reduced. The overall efficiency of the confinement scheme is also reduced by the energy losses in the supply system which cannot be reduced to a sufficiently low level because of the skin effect in the conductors.

Probably the most restriction on the plasma pressure in a practical device is imposed by power losses to the conducting wall. The feasible thermal load for copper (taken from gyrotron technology) $\lesssim 10^7$ W/m^2. Using this value for the same device we find the corresponding operational parameters, $n_e=10^{13}$ cm^{-3}, $T=1$ keV, $B=1.3$ kG, $f=1.4\cdot 10^7$ Hz, $V_p=27$ kV, $P\sim 20$ MW, $\tau_{ov}\sim 7.5\cdot 10^{-3}$ s, $P_h\sim 28$ kW. It is not clear how practical this voltage V_p is. Arcing in front of the gaps could impose limits of the voltages. The total power dissipated in the shell seems to be somewhat high for the machine of this size.

There might be a technical possibility to reduce this power loss by applying litz wire technology (see e.g. Ref. 23, and references therein) for the conducting shell. In this case the shell should be made of strands of thin individually insulated wires occupying the shell's volume such that the high frequency current (flowing in the skin layer of each wire) occupies a much larger volume than the skin layer of a solid conductor. Determination of feasibility of litz wire technology for this case requires separate study. If the use of this technology would reduce power losses by an order of magnitude, then the device with these parameters could be practical. If this technology is found to be effective it might be possible to achieve a fusion-grade plasma with $n_e=10^{13}$ cm^{-3} and $T=10$ keV in the compact configuration. The considered parameters of compact configurations approximately define the limits of a wide range of practical plasma parameters.

The parameters of a medium-sized device can be chosen as the following: $R_0 = 10$ m, $a = 0.5$ m, $n_e = 10^{13}$ cm^{-3}, $T = 100$ eV, $B = 400$ G, $f = 4.4 \cdot 10^5$ Hz, $V_p = 27$ kV, $P_s \sim 1.7 \cdot 10^5$ W/m^2, $P \sim 34$ MW, $\tau_{ov} \sim 2.4 \cdot 10^{-3}$ s, $P_h \sim 8.4$ MW. Possible use of the litz wire technology could improve the power requirements.

Even if the advanced technology is effective, the use of ordinary conductors for the shell's material leads to a very high power dissipation in the shell for fusion plasma parameters such that possible fusion reactor is not efficient. Probably the best that one can achieve (assuming litz wire technology is very effective) is that the fusion power is of the same order as power dissipated in the walls. Thus the use of superconducting materials is required for an efficient fusion reactor based on this confinement concept. In this case possible reactor parameters corresponding to 1000 MW fusion power are $R_0 = 20$ m, $a = 1$ m, $n_e = 10^{14}$ cm^{-3}, $T = 15$ keV, $B = 16$ kG, $f = 2 \cdot 10^6$ Hz, $V_p = 17$ MV, $\tau_{ov} \sim 2.8$ s, $P_h \sim 90$ MW.

IV. SUMMARY

Plasma confinement concept based on plasma confinement by electromagnetic pressure of circularly polarized rf field in the frequency range $f \ll c/a$ is proposed. A practical implementation of this concept in a toroidal device is suggested.

We demonstrated that nearly circular polarization of the electromagnetic field is required for plasma confinement from both the equilibrium and stability considerations. Numerical modeling is performed in 2D toroidal geometry to study the plasma confinement. The ideal MHD plasma model is used in the analysis with the assumption that there is a vacuum layer between plasma surface and the conducting shell. We found that within this model plasma is confined by the applied rf fields and its equilibrium is stable on the simulations time scale.

Based on the results of the modeling possible compact and medium size toroidal plasma confinement devices are proposed. The main limitation on the plasma pressure in these configurations is due to the Ohmic losses in the conducting shell. Because of this limitation possible application of this approach to the fusion reactor requires use of superconducting materials for the toroidal shell.

Our conclusions are based on a relatively simple plasma model used in the analysis. Thus the results only present a general idea of this plasma confinement concept. Further theoretical and experimental studies (including stability analysis in 3D geometry) are required for more reliable conclusions about the attractiveness of this concept. In particular, additional first-principles study is needed to address, in more detail, hot plasma effects on rf pressure and on plasma heating in rf fields.

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