LETTERS

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Alpha effect of Alfvén waves and current drive in reversed-field pinches

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Circularly polarized Alfvén waves give rise to an α-dynamo effect that can be exploited to drive parallel current. In a ''laminar'' magnetic the effect is weak and does not give rise to significant currents for realistic parameters (e.g., in tokamaks). However, in reversed-field pinches (RFPs) in which magnetic field in the plasma core is stochastic, a significant enhancement of the α effect occurs. Estimates of this effect show that it may be a realistic method of current generation in the present-day RFP experiments and possibly also in future RFP-based fusion reactors. © *1998 American Institute of Physics.* [S1070-664X(98)02803-1]

It has been long known,[1] that helical disturbances (''cyclonic events''[2]) of the magnetic field can give rise to an electric current parallel to the mean magnetic field. This so-called α effect[3] is believed to play an important role in the generation of cosmic magnetic fields (see, e.g., Ref. 4), although some doubt has been recently cast on the efficiency of this process.[5,6]

Typically, the α effect is associated with turbulent flows. However, as shown by Moffatt,[7] a single helical magnetohydrodynamic (MHD) wave can also give rise to the α effect. Injection of such waves has been suggested[8] as a possible current drive mechanism in tokamaks; initially interpreted to be a consequence of helicity conservation,[8] this current has been shown[9,10] to be due to the α effect of Alfvén waves. (In fact, a calculation based on the naive helicity conservation leads to the wrong conclusions,[10] although a nonzero wave helicity is required, at least within the confines of MHD; in more general theories this is not necessary[11]).

In practice, the α effect of Alfvén waves in a quasi-uniform magnetic field (such as the tokamak equilibrium field) is too weak to generate significant currents. However, as we argue in the present Letter, the presence of a small-scale magnetic structure can strongly enhance the wave α effect. This phenomenon is, in particular, relevant to reversed-field pinches (RFPs) for which the current drive is a major unresolved problem and in which the magnetic field possesses a significant stochastic component.

An Alfvén wave, with wave vector $\mathbf{k}$ and frequency ω in a uniform magnetic field $\mathbf{B}_0$, gives rise to the steady-state parallel electromotive force (emf) per unit length, given by

$$\mathscr{E} \equiv \frac{1}{4c}\langle \tilde{\mathbf{v}}^* \times \tilde{\mathbf{B}} \rangle \cdot \mathbf{b_0} + \text{c.c.} = \frac{k^2 D_R}{2k_\parallel c}\,\frac{i\tilde{\mathbf{B}}^* \times \tilde{\mathbf{B}} \cdot \mathbf{b}_0}{B_0}. \tag{1}$$

Here $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{B}}$ are the flow velocity and magnetic field of the wave, $\mathbf{b}_0 = \mathbf{B}_0/B_0$, $D_R = \eta c^2/4\pi$ is the resistive diffusion coefficient, η is resistivity, $k_\parallel \equiv \mathbf{k}\cdot\mathbf{b}_0$, the asterisk stands for complex conjugation, and $\langle\ \rangle$ denotes an average over the wave period.

The resulting parallel current J is determined from the mean parallel Ohm's law from which it follows that

$$J = \frac{\mathscr{E}}{\eta} = \frac{4\pi}{\eta B_0}\,\frac{k^2 D_R}{k_\parallel c}\,W, \tag{2}$$

for circularly polarized waves; here $W = |\tilde{\mathbf{B}}|^2/8\pi$ is the wave energy density. We can now compare the rf power P required to drive a certain current with the Ohmic power $P_o = \eta J^2$ required to drive the same current:

$$\frac{P}{P_o} = \frac{\omega}{k^2 D_R}\,\frac{k_\parallel R q}{Q} = S\left(\frac{k_\parallel}{k}\right)^2 \frac{q}{Q}, \tag{3}$$

where R is the major radius, $q = cB/4\pi JR$ is a measure of the safety factor, $Q = \omega W/P$ is the quality factor, $S \equiv v_A R/D_R$ is the Lundquist number, v_A is the Alfvén velocity, and we have exploited the Alfvén wave dispersion relation, $\omega = k_\parallel v_A$.

The magnitude of the quality factor Q that determines the relative current drive efficiency is given by the ratio of wave frequency and dissipation rate. If the latter were solely due to resistive dissipation in plasma, $Q = \omega/k^2 D_R = k_\parallel S/k^2 R$, so that $P/P_o \sim \mathscr{O}(1)$ for $k_\parallel R \sim 1$ and a typical tokamak with $q \sim 1$. The implied quality factor would be, however, very large: e.g., for present-day tokamaks, with $S = 10^{10}$, $k_\parallel/k = 0.1$, the required $Q \sim 10^8$. In real experiments, however, much lower values of $Q \lesssim 10^2 - 10^3$ are expected due to the existence of other wave energy loss channels (dissipation in resistive walls, plasma edge, etc.); consequently the wave power required to drive current would exceed the Ohmic power by a large factor.

Thus far, the equilibrium magnetic field was assumed to be quasi-uniform, not exhibiting any structure on the scale small compared to the transverse wavelength of the driven Alfvén wave. Consequently, the dynamo emf is small since, within the context of MHD, an Alfvén wave is only weakly damped by resistivity (with the damping rate $k^2 D_R/2$). In actual experiments, however, the magnetic field may exhibit a small-scale irregularity, due to, for example, tearing instabilities that can produce magnetic islands at mode-rational surfaces.

The presence of islands creates regions in which magnetic surfaces are destroyed. When islands overlap, magnetic field becomes stochastic,[12–14] as occurs, in particular, in the core of reversed-field pinches.

It has been argued that in the presence of a stochastic magnetic field Alfvén wave damping is enhanced.[15–17] Because of this increased dissipation the α effect can be expected to be enhanced. This phenomenon is most transparent in the context of the Similon and Sudan model[17] that we employ in the following discussion.

When magnetic surfaces are destroyed, the distance between neighboring field lines ξ grows exponentially with the distance s along the field [13,15]

$$\xi(s) = \xi_0 e^{s/\lambda_M}, \tag{4}$$

where ξ_0 is the initial field line separation and λ_M is the Lyapunov length. The magnitude of the Lyapunov length scale depends on the overlap parameter ζ defined as the ratio of the average size of the neighboring islands to their radial separation.[18] For islands with the same poloidal number,

$$\lambda_M = \frac{\pi R}{\ln(\pi\zeta/2)}, \tag{5}$$

and while it is unknown in the more general case, it can be expected to be smaller than the above value.[19] Thus, for strongly overlapping islands ($\zeta \gg 1$) a typical Lyapunov length is on the order of the torus major radius.

Let us now consider, following Similon and Sudan,[17] an Alfvén wave packet, with a characteristic perpendicular wave number k_0 ($\gg k_\parallel$), launched at $s=0$. Because the dispersion relation $\omega = k_\parallel v_A$ implies that wave group velocity is parallel to the magnetic field [in the absence of dispersive effects such as finite Larmor radius (FLR) corrections], the wave packet propagates along a magnetic flux tube, becoming distorted and filamented in the same manner as the flux tube cross section. The characteristic width of filaments decreases exponentially with s as follows from flux conservation and Eq. (4); consequently the characteristic transverse wave number grows exponentially with s:[17]

$$k_\perp(s) = k_0 e^{s/\lambda_M}. \tag{6}$$

The energy $\mathscr{E}$ of the wave packet decreases with distance, due to the resistive dissipation,

$$\frac{\mathscr{E}(s)}{\mathscr{E}(0)} = \exp\left(-\int_0^s ds' \, \frac{k^2(s') D_R(s')}{v_A(s')} \right). \tag{7}$$

The dissipation length λ_d is defined[17] as the distance at which the wave energy decreases by factor e^2:

$$\int_0^{\lambda_d} ds' \, \frac{k^2(s') D_R(s')}{2 v_A(s')} = 1, \tag{8}$$

i.e., as the length scale at which the resistive diffusion becomes strong. With the aid of Eq. (6) one then finds that

$$\lambda_d = \frac{\lambda_M}{2} \ln \frac{4S}{k_0^2 \lambda_M R.} \tag{9}$$

At this distance the transverse wavelength becomes

$$k_\perp(\lambda_d) = \left(\frac{4S}{\lambda_M R} \right)^{1/2}. \tag{10}$$

For typical experimental parameters, λ_d is several times larger than λ_M. For example, for present-day RFPs, with $S = 10^6$, $\lambda_M = R$ and $k_0 R = 10$, $\lambda_d \approx 5\lambda_M$. For such parameters, the implied quality factor $Q = \omega/2\gamma_d = k_\parallel \lambda_d/2$ (where the dissipation rate $\gamma_d = v_A/\lambda_d$) is small, $Q \approx 3$. The resistive dissipation (enhanced by the magnetic field stochasticity) would therefore be the main dissipation channel for Alfvén waves.

Because of the increase of the transverse wave number, the dynamo emf increases and becomes, according to Eqs. (1) and (10),

$$\mathscr{E} = \frac{2 v_A}{k_\parallel c \lambda_M} \frac{i \tilde{\mathbf{B}}^* \times \tilde{\mathbf{B}} \cdot \mathbf{b}_0}{B_0}. \tag{11}$$

Observe that the dynamo emf no longer depends on resistivity, but instead its magnitude is determined by the Lyapunov length of the stochastic magnetic field. For circularly polarized waves, it can be expressed as

$$\mathscr{E} = 8\pi \frac{\lambda_d}{\lambda_M} \frac{P}{k_\parallel c B_0}. \tag{12}$$

We can now proceed as previously and compare the powers required to drive a certain current Ohmically and by waves. This leads to

$$\frac{P}{P_0} = \frac{\lambda_M}{2\lambda_d} k_\parallel R q. \tag{13}$$

For present-day RFP parameters, mentioned earlier, with $k_\parallel R = 1$ and $q = 0.1$, this implies that $P/P_0 \sim 10^{-2}$.

It should be noted that the above result is valid only if $k_\perp \rho_i \ll 1$ (where ρ_i is the ion gyroradius) so that FLR corrections to the Alfvén wave dispersion relation can be neglected. This requires that the Lundquist number be sufficiently low:

$$S \ll \frac{\lambda_M R}{4\rho_i^2}. \tag{14}$$

For typical experimental parameters, the above inequality is not satisfied and therefore the FLR effects need to be included. (The contribution of resistivity can be neglected since it is small compared to that of the FLR).

The FLR effects modify the Alfvén wave dispersion relation which for $\omega \ll k_\parallel v_{te}$ and $k_\perp \rho_e \ll 1$ (v_{te} and ρ_e are electron thermal velocity and gyroradius, respectively) and equal electron and ion temperatures become[20]

$$\omega = k_\parallel v_A \Gamma(\mu), \tag{15}$$

where $\mu = k_\perp \rho_i$ and

$$\Gamma(\mu) = \mu \left(\frac{1}{1 - e^{-\mu^2} I_0(\mu^2)} + 1 \right)^{1/2}. \tag{16}$$

The $k_\perp$ dependence gives rise to the transverse component of group velocity so that the wave packet spreads while propagating along the flux tube. Consequently the evolution of $k_\perp$ is described by

$$\frac{d\mu}{ds} = \frac{\mu}{\lambda_M} - k_\parallel \mu^2 \Gamma'(\mu), \tag{17}$$

where the prime denotes the derivative. As the wave packet propagates, $k_\perp$ increases until the right-hand side of Eq. (17) vanishes, which occurs when

$$\frac{1}{k_\parallel \lambda_M} = \mu \Gamma'(\mu). \tag{18}$$

For $k_\parallel \lambda_M \sim 1$, $\mu \sim 1$. In particular, for parameters of the Madison Symmetric Torus (MST) reversed-field pinch for which $S = 10^6$, $\lambda_M = 1$ m, $R = 1.5$ m and $\rho_i = 1$ cm,[21] and assuming $k_\parallel R = 1$, Eq. (18) implies that $k_\perp = 1.2\ \mathrm{cm}^{-1}$. Consequently, the quality factor $Q \approx 40$ due to the resistive dissipation is within the expected range mentioned earlier so that the ratio of required rf and Ohmic powers given by Eq. (3) is $P/P_o \approx 0.1$.

While the above result seems quite attractive, the dependence on the Lundquist number implied by Eq. (3) indicates that the efficiency will be much lower in a hypothetical RFP fusion reactor. Assuming that such a reactor would have parameters $R = 3$ m, $T = 10$ keV, $B = 30$ kG, $n = 10^{14}\ \mathrm{cm}^{-3}$, and $q = 0.1$, for which $S \approx 10^{10}$, the implied $Q \sim 10^4$ due to the resistive dissipation and for $k_\parallel R = 1$ exceeds the expected upper value; assuming therefore $Q = 100$, one finds that $P/P_o \approx 10$.

Thus, in a reactor the current drive efficiency is lower than Ohmic; nevertheless, it should be noted that it is significantly higher than, e.g., the relative current drive efficiency by lower hybrid waves in a tokamak reactor. As argued by Boozer,[22] in the latter case $P/P_o \approx 10^3$.

Summarizing, we have suggested that the presence of a background of a stochastic magnetic field can enhance the α effect of Alfvén waves. While we found that inclusion of FLR effects significantly reduces the magnitude of the induced electromotive force, the latter can nevertheless generate currents with efficiency comparable to or greater than Ohmic for parameters of existing RFP experiments. For parameters of a hypothetical fusion reactor, this current drive efficiency is lower than the Ohmic by approximately an order of magnitude, but it exceeds the similar relative efficiency of lower hybrid current drive in a tokamak reactor by one to two orders of magnitude.

Our discussion was rather heuristic and it did not take, in particular, into account the effect of wave-driven current on the equilibrium magnetic field, This question needs to be addressed, especially in the situations when all parallel current is generated by waves. For the purpose of application to a RFP reactor, it should also be resolved whether the level of magnetic field stochasticity required for an efficient current drive can be made consistent with requirements on heat transport.

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