Mass-Dependent Ion Heating during Magnetic Reconnection in a Laboratory Plasma

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Noncollisional ion heating in laboratory and astrophysical plasmas and the mechanism of conversion of magnetic energy to ion thermal energy are not well understood. In the Madison Symmetric Torus reversed-field pinch experiment, ions are heated rapidly during impulsive reconnection, attaining temperatures exceeding hundreds of eV, often well in excess of the electron temperature. The energy budget of the ion heating and its mass scaling in hydrogen, deuterium, and helium plasmas were determined by measuring the fraction of the released magnetic energy converted to ion thermal energy. The fraction ranges from about 10%–30% and increases approximately as the square root of the ion mass. A simple model based on stochastic ion heating is proposed that is consistent with the experimental data.

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Ions are frequently heated rapidly during magnetic reconnection in laboratory and astrophysical plasmas. For example, strong ion heating is observed in tokamaks [1,2], merging plasma experiments [3,4], reversed-field pinches [5–9], linear magnetic mirrors [10], and in the solar corona [11–14]. In the Madison Symmetric Torus (MST) reversed-field pinch experiment [15] ions are heated during impulsive reconnection events (also known as sawtooth crashes), attaining temperatures exceeding hundreds of eV, often well in excess of the electron temperature [6,8,9]. The energy source for this heating is the equilibrium large-scale magnetic field that confines the plasma, but the means by which magnetic energy is converted to kinetic energy is yet to be determined.

In this Letter we investigate the energy budget of the reconnection heating by measuring the fraction of the released magnetic field energy that is converted into majority ion thermal energy. While previous measurements established that heavier ions are heated more strongly than lighter ions [9], the energy budget was not evaluated, in part because many of these measurements were for impurity (minority) ions. Analyzing plasmas with different majority ions allows the energy budget to be determined more accurately, revealing the ion mass dependence.

The MST is a toroidal device with a major radius $R = 1.5$ m and a minor radius $a = 0.51$ m. The majority ion temperature was measured using Rutherford scattering (RS) [16] of a beam of He atoms with an energy of 17 keV injected into the MST plasma. A small fraction of injected atoms undergoes scattering with the plasma majority ions, and by measuring the energy spectrum of the scattered atoms, the temperature of the majority ions can be determined. The spatial resolution of the diagnostic, determined by the intersection volume of the beam and the line of sight of the particle analyzer, is 4 cm in the toroidal and poloidal directions (perpendicular to the beam) and 14 cm in the radial direction. The direction of the beam-atom scattering into the diagnostic's detector is such that only the perpendicular ion temperature can be measured. The temporal resolution, limited by the count rate, was 10 μs. This is one of the few diagnostics that is able to provide the majority ion temperature in a high-temperature plasma, and the MST is one of the few fusion devices that currently employs such a diagnostic.

The time evolution of the deuterium ion temperature and equilibrium magnetic field energy during reconnection is shown in Fig. 1. The ion heating rate is about 2 MeV/s. The time evolution of hydrogen, deuterium, and helium ion temperatures, measured at the RS sample volume centered at $r = 15$ cm, is shown in Fig. 2. These measurements illustrate the fact that the heavier ions are heated more strongly. Similar conclusions have been drawn from observations of ion heating in the solar corona [14].

The fraction of released magnetic field energy that appears as ion thermal energy can be defined as the ratio of the change in the ion thermal energy to the change in the magnetic field energy: $\Delta E_{\mathrm{therm}}/\Delta E_{\mathrm{mag}}$, where E_{therm} and E_{mag} are determined from two volume integrals,

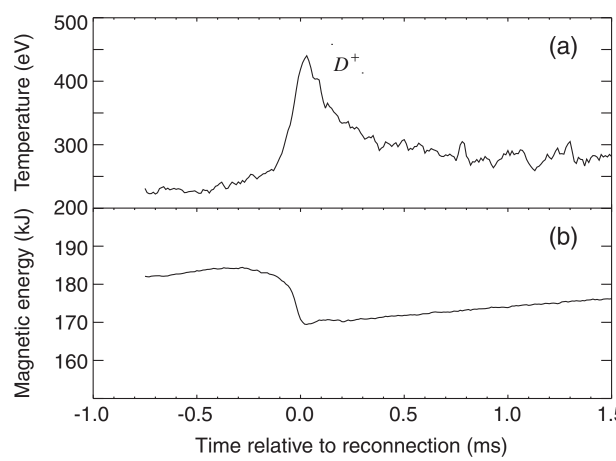

FIG. 1. Time evolution of (a) D^+ ion temperature measured at $r = 15$ cm and (b) magnetic field energy during reconnection.

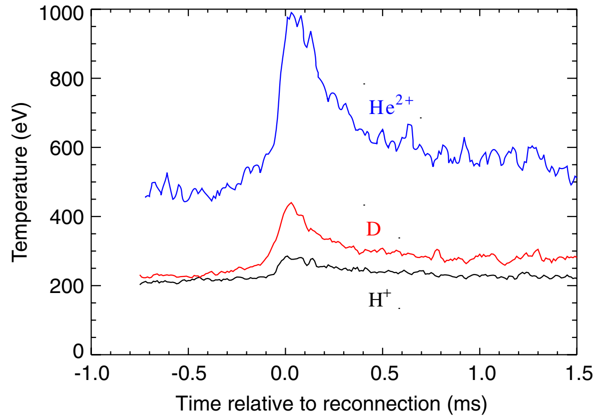

FIG. 2 (color online). Comparison of reconnection heating in hydrogen, deuterium, and helium plasmas. The ion temperatures are measured at $r = 15$ cm.

$$E_{\mathrm{therm}} = \int \frac{3}{2} kT_i \frac{n_e}{Z_i} dV, \qquad E_{\mathrm{mag}} = \int \frac{B^2}{2\mu_0} dV. \quad (1)$$

The magnetic field energy was calculated using equilibrium field reconstructions which take into account the plasma current and pressure, and the magnetic field at the plasma boundary. The plasma electron density profile $n_e(r)$ was measured using an 11-chord interferometer [17], and the majority ion density was calculated as $n_i = n_e/Z_i$, where Z_i is the ion charge. The electron temperature, measured with Thomson scattering, was hundreds of eV, so the ions under examination are fully stripped of electrons. To measure the ion temperature profile the sample volume of the RS analyzer was moved radially between discharges. Although the RS radial resolution is fairly coarse, the ion thermal energy is well measured, because the ion temperature profile tends to be flat [9].

The mass dependence of the fractional energy transfer is shown in Fig. 3. The least-χ^2 fit (shown with a continuous line) is close to $\propto M_i^{1/2}$, and the dependence on the density,

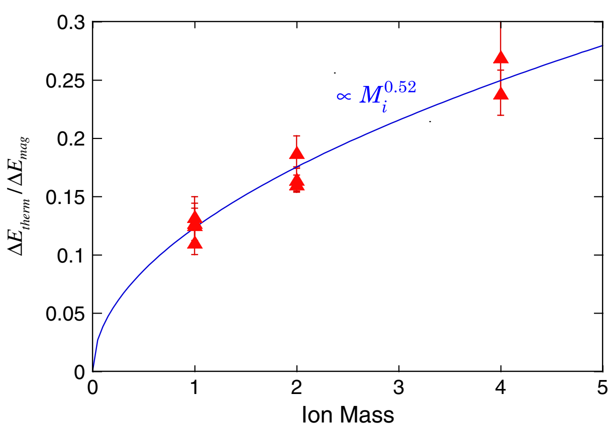

FIG. 3 (color online). Fraction of magnetic energy thermalized into ions $\Delta E_{\mathrm{therm}}/\Delta E_{\mathrm{mag}}$ for H^+, D^+, and He^{2+} ions. Different data points at the same mass correspond to different plasma densities in a range $(0.8\text{–}1.4) \times 10^{19}$ m^{-3}.

which is shown as different data points at the same mass, is weak. Although the data on the ion charge dependence are very limited (only H^+, D^+, and He^{2+} are available), it appears to be weak as well. Ion energy losses and temperature isotropization during the heating burst do not significantly affect the result, because the heating rate is much faster than these processes. For example, the equilibration time between $T_\perp$ and $T_\parallel$ is 4 ms for 200 eV D^+ ions. The impact of these effects can be estimated from the temperature decay after the reconnection at relative time $t > 0$. Inclusion of the loss rate, based on this estimate, changes the mass dependence from $M_i^{0.52}$ to $M_i^{0.54}$.

There are a number of mechanisms proposed to explain noncollisional ion heating, but there has been no satisfactory explanation consistent with all the experimental observations. For example, the viscous damping of tearing flows [18] requires a highly sheared fast ion flow that has not been observed in experiments. Ion heating via ion-cyclotron damping of an Alfvénic cascade [19] can be effective in principle, but it depends strongly on the charge, mass, density (collisionality), and also the fluctuation amplitude, which can be small at the ion-cyclotron range of the power spectrum. Stochastic ion heating, considered in [20], relies on polarization drift to traverse ions across a short-wavelength region of electrostatic-drift-Alfvén fluctuations. This effect is estimated to be small in the MST.

A simple model of stochastic ion heating is suggested here that accounts for the fast rate of ion heating, the mass dependence, as well as other previously measured data relevant to ion heating. We propose that a large cross-field radial transport of ions through a strong fluctuating radial electric field causes fast random changes in the ion perpendicular $E \times B$ drift velocity, which results in perpendicular heating. The cross-field radial transport is caused by magnetic stochasticity generated by multiple tearing instabilities that attain especially large amplitudes during the fast reconnection events.

Suppose that there is a fluctuating radial electric field with an amplitude E_r, which creates a perpendicular drift velocity $v_E = E_r/B_0$, where B_0 is the large-scale mean magnetic field (assumed uniform for simplicity). Consider that the electric field has a fine radial structure with a radial correlation length δ_r. Consider also that an ion experiences radial cross-field excursions due to wandering stochastic magnetic field lines. Then the drift velocity will be randomly changing with an average time step τ, which can be estimated as the time it takes an ion to traverse the radial correlation length δ_r. For example, if the cross-field ion motion is diffusive, characterized by a diffusion coefficient $D_\perp$, then the decorrelation time can be estimated as $\tau \approx \delta_r^2/D_\perp$. The perpendicular velocity diffusion coefficient can be estimated as $D_v = v_E^2/\tau$ and, applying the standard random walk arguments, the time evolution of the perpendicular energy is $M_i v_\perp^2/2 \approx M_i D_v t/2 = M_i v_E^2 t/2\tau$, where M_i is the ion mass. Combining the above equations, one can write an estimate for the perpendicular heating rate:

$$\gamma_\epsilon = \frac{1}{v_{Ti}^2}\frac{\partial v_\perp^2}{\partial t} = \frac{E_r^2 D_\perp}{\delta_r^2 B_0^2 v_{Ti}^2}, \quad (2)$$

where v_{Ti} is the ion thermal velocity (10^5 m/s for a 200 eV D^+ ion).

Measurements of power spectra and two-point spatial correlation measurements of electrostatic fluctuations in MST, made with electrostatic Langmuir probes at $r = 40$ cm, have shown that the amplitude of the potential fluctuation is high $\tilde{\varphi} \approx 60$ V ($e\tilde{\varphi}/T_e \sim 1$), and the radial electric field reaches $E_r \approx 6$ kV/m. The power spectrum is broad, with most of the fluctuation power contained in the low-frequency range below 50 kHz; Fig. 4(a). The cross-coherence between two spatial points separated radially by 2 cm drops to the noise level at $f \gtrsim 5$ kHz as seen in Fig. 4(b). Therefore, at frequencies higher than 5 kHz electrostatic fluctuations have a radial correlation length less than 2 cm while still sustaining a significant power. The parallel and the perpendicular wavelengths are greater than the radial correlation length. These results are in a good agreement with previous measurements of electrostatic fluctuations in various reversed-field pinch devices [21–24]. The plasma parameters for the experiment described in this Letter were chosen to match past experiments in order to use quantities previously measured in similar plasma conditions.

Finally, cross-field radial transport in the MST strongly increases during reconnection; measurement of the radial particle diffusion [25] indicates that, at the peak of a sawtooth crash, the particle diffusion coefficient reaches $D_\perp \approx 10^3$ m^2/s.

To estimate the perpendicular heating rate given by Eq. (2) assume $E_r = 6 \times 10^3$ V/m, $\delta_r = 0.03$ m, $D_\perp = 10^3$ m^2/s, $v_{Ti} = 10^5$ m/s, and $B_0 = 0.3$ T. This results in $\gamma_\epsilon \approx 4 \times 10^4$ s^{-1}, close to the value observed in MST ($\sim 10^4$ s^{-1}). Hence, stochastic heating can be potentially very strong, even after allowing for reasonable uncertainties of the parameters in Eq. (2).

In order to further explore this mechanism and to evaluate the ion distribution function during stochastic heating, let us consider a simple model of particle motion in crossed electric and magnetic fields and simulate the effect of random cross-field motion by introducing a stochastic, time dependent phase of the radial electric field. Assume a uniform magnetic field B_0 in the z direction and an electric field E in the x direction. The equations of motion of a test ion with a mass M_i and a charge q_i are

$$M_i\frac{dv_x}{dt} = q_i E + q_i v_y B_0, \qquad M_i\frac{dv_y}{dt} = -q_i v_x B_0. \quad (3)$$

To model the ion motion through a radial electrostatic field with a finite correlation length δ_r, assume, for simplicity, that the electric field is constant in space and time, except that it changes its magnitude (and direction) at a random time series $t = t_1, t_2, \ldots, t_i, \ldots$ according to $E(t_i) = E_0\cos(\phi_i)$. The average time interval between the phase change events is chosen as $\overline{t_{i+1} - t_i} = \tau = \delta_r^2/D_\perp \approx 1$ μs ($\delta_r = 3$ cm and $D_\perp = 10^3$ m^2/s), and the phase is a random number uniformly distributed between $-\pi$ and π. The electric field uniformity assumption is justified because the ion gyroradius is smaller than the spatial scales of the electrostatic fluctuations $k_r\rho_{ci} < 1$, and $k_\perp\rho_{ci}$, $k_\parallel\rho_{ci} \ll 1$.

The calculated time dependence of the perpendicular temperature $kT_\perp = \langle\varepsilon\rangle = \langle M_i(v_x^2 + v_y^2)/2\rangle$ is shown in Fig. 5(a) (triangles) for $E_0 = 6$ kV/m, $B_0 = 0.3$ T, $M_i = 2$. The brackets denote averaging over 10 000 particles launched with a zero initial perpendicular energy. The continuous line shows the theoretical expectation $\langle\varepsilon_\perp\rangle = M_i v_E^2 t/2\tau$. The ion velocity distribution function is shown in Fig. 5(b), and it is very well fit with a Gaussian distribution (shown in the continuous line). The fitted ion tem-

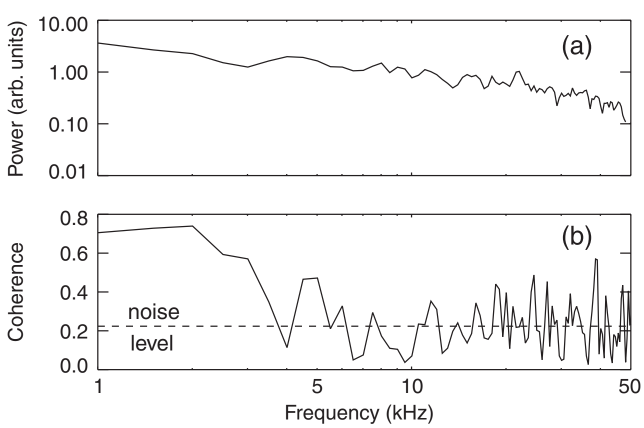

FIG. 4. Electrostatic fluctuations in MST. (a) Power spectrum measured at 11 cm inside the plasma boundary. (b) Cross-coherence between two signals separated radially by 2 cm. The statistical noise level is indicated by a dashed line.

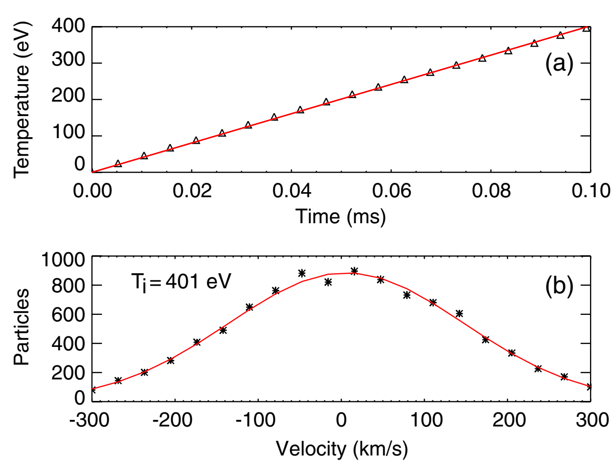

FIG. 5 (color online). Stochastic simulation of perpendicular energy gain. (a) $T_\perp$ versus time. Data points, simulation; continuous line, theory. (b) Ion energy distribution function at $t = 0.1$ ms. Continuous line, Gaussian fit with $T_i = 401$ eV.

perature $T_i = 401$ eV, with a heating rate of 4 MeV/s, which is within a factor of 2 of the measurement.

The stochastic ion heating process suggested in this Letter yields a large energy growth consistent with that observed in the experiment. The mechanism relies on a synergy of electrostatic and magnetic fluctuations: both are required to be present in order for the heating to occur. The main contribution of the electrostatic fluctuations comes from the low-frequency range ($5 \lesssim f \lesssim 50$ kHz $\ll f_{ci}$), which contains most of the fluctuation power. Note that magnetic fluctuations appear implicitly through the radial stochastic diffusion. The strength of the diffusion depends not only on the amplitude of the tearing modes but also on the characteristics of the tearing spectrum. For example, the absence of edge-resonant tearing modes inhibits nonlinear mode coupling, magnetic reconnection, and radial transport [26,27]. Consequently, the ion energy gain is expected to be small as well, which is consistent with the well-known observation that the reconnection-based ion heating in the MST is very weak without edge-resonant tearing modes even when the core-resonant modes are large [9].

In addition, the following results from the model are also consistent with the experimental observations in the MST. First, because the drift velocity does not contain the electric charge, stochastic ion heating should not depend on the charge. Second, if the fluctuation amplitude is mass-independent and the diffusion coefficient $D_\perp \propto M_i^{-1/2}$ [28], then the energy gain predicted by Eq. (2) is proportional to $M_i^{1/2}$. Analogously, the electron energy gain is predicted to be very small due to the electron's low mass.

Mass (and charge) dependent ion heating, albeit with a different mass scaling, is also observed in nonterrestrial plasmas, e.g., the solar corona and solar wind [14]. The difference may result from the fact that the nonterrestrial measurements were based on trace ion species, while the measurements described here are for the majority species. But it may also indicate that there are simply differences in the physics of ion heating in laboratory and nonlaboratory plasmas and that the model presented here is not universally applicable.

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