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# Identification of island-induced Alfvén eigenmodes in a reversed field pinch

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**Abstract**

The modification of the shear Alfvén spectrum due to a core resonant magnetic island is used to explain the Alfvénic activity observed on the Madison symmetric torus (MST) reversed-field pinch during neutral beam injection. Theoretical studies show that the Alfvén continua in the core of the island provide a gap in which the observed Alfvénic bursts reside. Numerical simulations using a new code called SIESTAlfvén have identified the bursts as the first observation of an island-induced Alfvén eigenmode (IAE) in an RFP. The IAE arises from a helical coupling of harmonics due to the magnetic island.

Keywords: Alfvén, MHD, magnetic island, MST

(Some figures may appear in colour only in the online journal)

## Introduction

Understanding the spectrum of shear Alfvén waves in a magnetically-confined plasma is crucial for determining the stability properties of the configuration. Frequency gaps in the continuous spectrum (continuum) can lead to discrete Alfvén modes that do not experience continuum damping and can be destabilized by energetic particles such as those driven by neutral beam injection (NBI) or the copious fast $\alpha$-particles present in a fusion reactor. Alfvén instabilities are a general feature of plasma equilibria and can occur in tokamaks, stellarators, and reversed-field pinches (RFPs). Some common Alfvén modes that can be driven unstable include the toroidicity-induced Alfvén eigenmode (TAE) and the beta-induced Alfvén eigenmode (BAE). These modes lie in gaps that arise from a coupling of poloidal mode numbers due to toroidicity and from finite shear Alfvén wave compressibility via geodesic curvature coupling, respectively [1, 2].

In the EAST, FTU, and TEXTOR tokamaks, BAE activity has been observed in the presence of a magnetic island [3, 4]. The frequency of these Alfvén eigenmodes (AEs) varies strongly with the tearing mode amplitude (island width), suggesting that an island can affect the Alfvén continua and modes. Additionally, an Alfvén eigenmode has been identified on the TJ-II stellarator during tearing mode activity due to the island's helical coupling [5]. This manuscript presents the first explanation of an Alfvénic mode observed in an RFP as an island-induced Alfvén eigenmode (IAE). The IAE is shown to arise due to the helical coupling present in the core of a magnetic island. The mode identified as the IAE is observed on the Madison Symmetric Torus (MST) during NBI.

Alfvénic activity has been observed in the Madison Symmetric Torus RFP experiment during neutral beam injection [6]. Several different NBI-driven modes have been observed including a mode with $n = 5$ toroidal mode number and another mode with a dominant $n = 4$. The $n = 5$ mode has been identified as an energetic particle mode (EPM) and will not be discussed in detail here. The $n = 4$ mode has been identified as an AE due to its experimental scaling with the Alfvén speed. Density fluctuations associated with the $n = 4$ mode are measured to be core-localized [7]. Additional $n = -1$ activity is also observed around the same time as the $n = 4$ activity on MST. A characterization of the specific taxonomy for the beam-driven AE has not been determined, and will be the subject of this work.

 

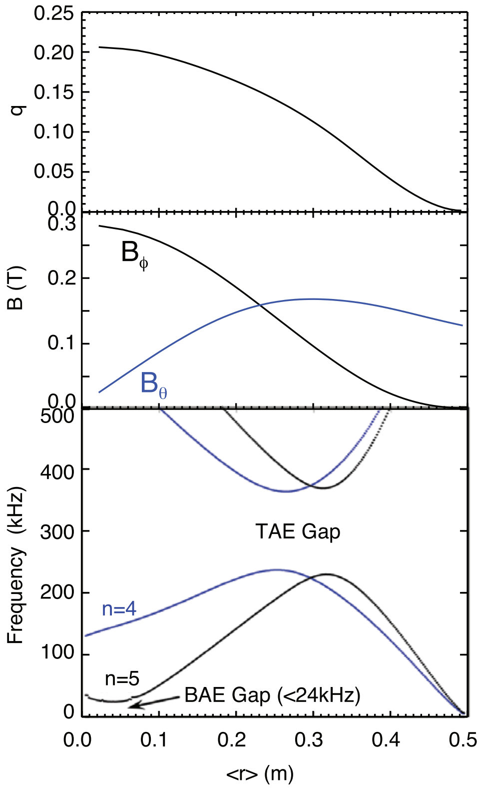


**Figure 1.** Safety factor and magnetic field profiles for a non-reversed 300 kA MST configuration. The bottom figure shows the shear Alfvén continua computed using STELLGAP with finite beta effects included through the slow sound approximation. The BAE gap exists below 24 kHz, and the TAE gap is between 250 and 350 kHz.

## Observations of Alfvénic activity in MST

MST is an axisymmetric RFP device with a close-fitting conducting shell. MST has a major radius of $R_0 = 1.5$ m and a minor radius of $a = 0.5$ m. Typically, plasma currents in MST are in the range of 200–500 kA [8]. The geometry in MST is amenable to a cylindrical approximation as toroidal effects are relatively weak on MST due to the small safety factor $q < 1$. In addition, the circular cross section removes much of the coupling that would be present in tokamaks, which typically have stronger non-circular shaping.

In this manuscript, MST is studied in the non-reversed configuration where the toroidal field vanishes at the edge. The $q$ profile and magnetic field for this configuration are presented in figure 1. The $q$ profile is small and monotonically decreases with radius and becomes approximately $q = 0$ at the edge. This profile is unstable to a number of tearing instabilities. The core region of this configuration is dominated by a tearing mode-induced magnetic island at the $q = 1/5$ surface with mode numbers $m_0 = 1$ and $n_0 = 5$.

A 1 MW neutral beam injector is installed on MST for energetic particle studies in the strongly sheared, high-beta RFP environment [9]. The NBI can fire hydrogen or deuterium beams at energies up to 25 keV, corresponding to beam velocities of $2.2 \times 10^6$ m s$^{-1}$ for hydrogen and $1.6 \times 10^6$ m s$^{-1}$ for deuterium. The beam is injected at an angle tangential to the core magnetic field. The pulse length of the beam is

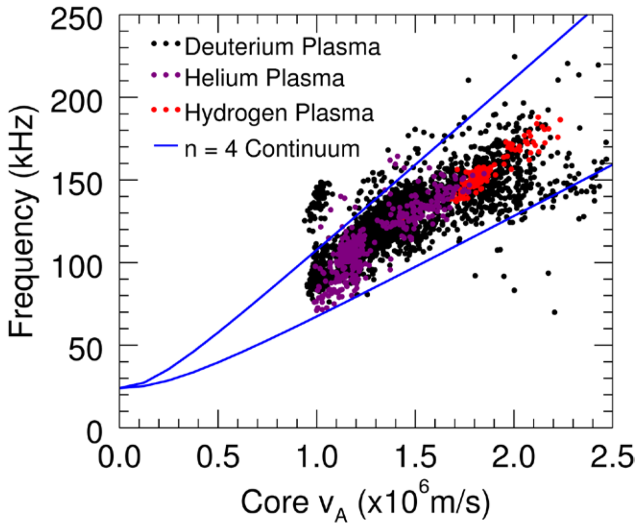


**Figure 2.** The theoretical continuum frequencies in the island core for $j_{\text{in}} = 1$ and $j_{\text{in}} = 2$ plotted along with the observed Doppler-corrected Alfvénic burst data from MST.

$\tau_{\text{beam}} \sim 5$–20 ms and can be considered a steady-state component for an analysis of Alfvén waves during tearing mode activity, since both the Alfvén timescale $\tau_A = 1/(k_\| v_A) \sim 1\ \mu$s and the island toroidal rotation timescale $\tau_{\text{rot}} = 1/f_{\text{rot}} \sim 250\ \mu$s are much shorter than the beam duration, $\tau_A \ll \tau_{\text{rot}} \ll \tau_{\text{beam}}$.

The fast ion population injected by the neutral beam serves as a destabilizing drive for energetic particle modes and Alfvén eigenmodes. By scanning the plasma current from $I_p \approx 200$ kA to $I_p \approx 500$ kA, the core magnetic field strength is adjusted, resulting in a change in the Alfvén speed, $v_A = B/\sqrt{\mu_0 \rho}$. A detailed scan in observed burst frequency versus core Alfvén speed has been performed. The magnetic activity at $\geqslant$100 kHz on MST is measured using 32 $B_\theta$ signals from a toroidal array of magnetic coils and 8 $B_\theta$ and $B_\phi$ signals each from a poloidal array of coils. Using these arrays, the mode of interest has been found to have dominant poloidal mode number $m = 1$ and dominant toroidal mode number $n = 4$. The results of the scan for deuterium, hydrogen, and helium plasmas are presented in figure 2. These frequencies have all been Doppler-corrected for the instantaneous plasma toroidal rotation frequency of 10–30 kHz. The $n = 4$ bursts are clearly AEs as their frequencies exhibit a strong scaling with core Alfvén speed. Additional $m = 0$, $n = -1$ activity is also measured on MST at the same time as the $n = 4$ bursts [7].

## Modeling and theory results

Since the $n = 4$ activity is Alfvénic, the question naturally turns to identifying the taxonomy of the AE. Initial simulations were performed to identify the TAE and BAE gaps in MST numerically using the VMEC and STELLGAP codes [10]. VMEC is a three-dimensional ideal MHD equilibrium code that solves for equilibria with closed nested flux surfaces [11, 12]. STELLGAP computes the shear Alfvén continua for the resulting equilibrium in the low $\beta$ limit [13].

The Alfvén continua for core $v_A = 1.75 \times 10^6$ m s$^{-1}$ is calculated using the slow sound approximation ($\gamma p/\rho\omega^2 R_0^2 \ll 1$) that includes acoustic couplings to lowest order [14], and is presented in the bottom plot of figure 1. The continua is calculated using a VMEC equilibrium and the STELLGAP code. Both the TAE gap (250–350 kHz) and the BAE gap (0–24 kHz) fall well outside the observed frequency range, so the Alfvénic activity on MST is neither a TAE nor a BAE.

These simulations fail to identify the $n = 4$ AE. The VMEC and STELLGAP computations assumed closed, nested flux surfaces with no islands. While a number of tearing instabilities are known to be present in MST during the time of the Alfvénic activity, it is known that a sizable 1/5 island dominates the core region of the non-reversed configuration. In particular, Thomson scattering fluctuations correlated with edge-measured magnetic amplitudes show a clear $n_0 = 5$ island structure [15, 16]. The $m_0 = 1$, $n_0 = 5$ island has a half-width of about 7 cm. It is natural to investigate whether the inclusion of the island could affect both the Alfvén continuum and discrete modes in MST.

The Alfvén continua in the presence of an island has been studied previously [17–20]. These studies show that the island causes a minimum continuum accumulation point frequency upshift; the location of the minimum frequency moves from the rational surface to the separatrix. Additionally, the helical coupling of the island is shown to create a continuum gap region centered on the island core. The local maximum continuum frequencies that bound the gap derived in [20] can be written in a form more amenable to comparison with experiment:

$$f = \sqrt{f_{\mathrm{BAE}}^2 + \frac{n_0^2 j_{\mathrm{in}}(j_{\mathrm{in}} + 2)}{4}\epsilon^2 k_\parallel^2 v_A^2}. \quad (1)$$

Here $k_\parallel = 1/(q_0 2\pi R_0)$, $\epsilon = q'_0 w/2$ is the normalized island half-width, and $j_{\mathrm{in}}$ is an integer that denotes the branch of the continua. $f_{\mathrm{BAE}}$ is the minimum BAE frequency at the rational surface in the absence of the island.

For MST, the characteristic parameters are $n_0 = 5, q_0 = 1/5$, $R_0 = 1.5$ m, $k_\parallel = 0.53$ m$^{-1}$, $f_{\mathrm{BAE}} = 24$ kHz (from figure 1), $w/2 = 7$ cm, $q'_0 = .004$ cm$^{-1}$, and $\epsilon = q'_0 w/2 = .028$. When these parameters are used in equation (1), the resulting continuum frequencies as a function of Alfvén speed for $j_{\mathrm{in}} = 1$ and $j_{\mathrm{in}} = 2$ are plotted in figure 2 along with the data. Experimental data is available in the range of $v_A \approx 1 \times 10^6$ m s$^{-1}$ to $2.5 \times 10^6$ m s$^{-1}$. The burst data tend to lie inside of the envelope formed by the $j_{\mathrm{in}} = 1$ and $j_{\mathrm{in}} = 2$ continuum branches from theory. Thus, the experimental observations of an Alfvén eigenmode are consistent with a mode existing in the frequency gap induced in the core of an $n_0 = 5$ island. This gap is named here the island-induced Alfvén eigenmode (IAE) gap, since this gap in the continuous spectrum is caused from the helicity of the island.

In order to verify the existence of a discrete island-induced mode, the Alfvén modes in an equilibrium containing an island must be computed. For this purpose the SIESTAlfvén code has been created [21]. SIESTA is a three-dimensional MHD equilibrium code capable of resolving magnetic islands

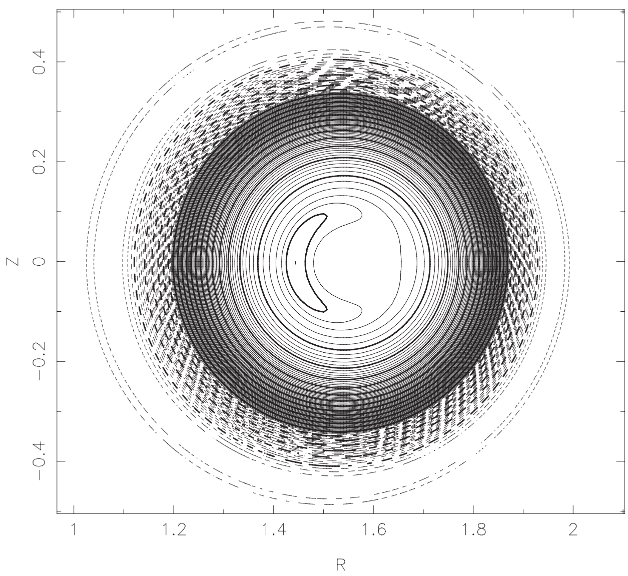


**Figure 3.** Contours of constant pressure in a poloidal plane at constant $\zeta$ from an MST SIESTA equilibrium. The $m_0 = 1$ character of the island is clearly visible.

[22]. The SIESTA iterative scheme works as follows: beginning with an axisymmetric VMEC equilibrium, non-ideal resonant perturbations are added to break the ideal flux surfaces at chosen rational surfaces. Next, the nonlinear ideal MHD energy is minimized by solving the linearized ideal MHD force balance equations to find a new plasma displacement $\boldsymbol{\xi}$:

$$H_{ij}\xi^j = -F_i. \quad (2)$$

Here $H_{ij} = \partial F_i/\partial \xi^j$ is the Hessian matrix for the iteration and the covariant components of the current nonlinear force residual are given by

$$F_i = \epsilon_{ijk}\sqrt{g}\, J^j B^k - \frac{\partial p}{\partial x^i}, \quad (3)$$

where the coordinate $x^i \in \{r, u, v\}$ and $\sqrt{g}$ is the Jacobian. After each step, the new nonlinear force is obtained and compared to the specified force tolerance. If it is above the tolerance, the process is repeated. If it is below the tolerance, an equilibrium solution has been found.

SIESTAlfvén is a new post-processing code, which computes Alfvén eigenmodes and is initialized by a converged SIESTA equilibrium, with $F_i \to 0$. An inertial term is added to equation (2),

$$-\omega^2 \rho g_{ij}\xi^j = H_{ij}\xi^j, \quad (4)$$

to form the eigenmode equation SIESTAlfvén uses to compute the eigenmodes and eigenfrequencies. Here $\rho$ is the plasma mass density and $g_{ij}$ are the lower metric elements for the SIESTA coordinates. This is the MHD eigenmode equation that SIESTAlfvén uses to find the modes and corresponding frequencies. SIESTAlfvén solves equation (4) using a windowed eigensolver, allowing for a targeted eigenvalue range to be specified [23]. The eigenmode solutions are then analyzed. The actual shear Alfvén surface component $\xi_{\mathrm{surf}}$ is computed from the radial, poloidal, and toroidal components

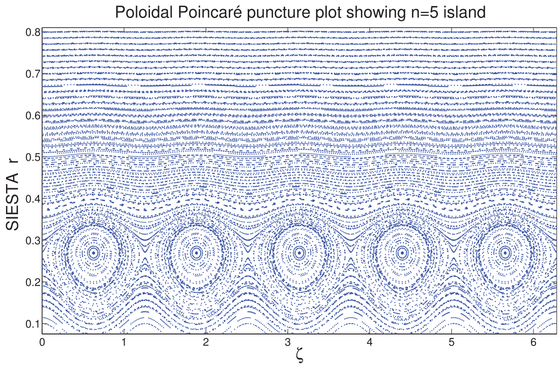


**Figure 4.** Poincaré puncture plot from the SIESTA magnetic field intersecting a toroidal plane at constant $\theta$. Punctures are shown for $r = \sqrt{\psi}$ versus $\zeta$. The $n_0 = 5$ mode number of the island can be seen.

as a post-processing step to the eigenmode solve. The surface component is computed as $\xi_{\text{surf}} = \boldsymbol{\xi} \cdot (\mathbf{B} \times \nabla p)/|\mathbf{B} \times \nabla p|$, where we are assuming the pressure profile equilibrates on magnetic surfaces.

The non-reversed MST configuration with an island and a plasma current of $I_p = 300$ kA has been reproduced using the SIESTA code. A reconstructed axisymmetric VMEC equilibrium was used to initialize the SIESTA code. The SIESTA reconstruction results in an equilibrium with an island that retains the original magnetic axis. For this reconstruction, only $n_0 = 5$ activity is allowed to break the axisymmetric topology. While other tearing modes are present in the experiment, the predicted IAE is localized within the dominant $m_0/n_0 = 1/5$ island and is therefore insensitive to other tearing activity. The pressure contours from the converged SIESTA equilibrium are shown in figure 3. The surfaces of constant pressure are plotted in $R$, $Z$ space. Near the magnetic axis, a large island is present with poloidal mode number $m_0 = 1$. The island width is about 15 cm and has a bean-shaped appearance, consistent with experimental reconstructions from the NCT code. Figure 4 displays the Poincaré puncture plots for the SIESTA magnetic field through a surface of constant poloidal angle, $\theta$. The punctures are plotted for toroidal angle $\zeta$ versus SIESTA radius $r = \sqrt{\psi}$, where $\psi$ is the normalized poloidal flux. In this plane, the toroidal mode number $n_0 = 5$ is clearly visible.

SIESTAlfvén is used to investigate the MHD modes present in MST, resulting in the discovery of a discrete mode at 145 kHz. The mode surface displacement is plotted in figure 5. The mode is dominated by the $m = 1$, $n = 4$ and $m = 0$, $n = -1$ harmonics, with peaks centered at the core of the magnetic island extending from about $r = 0.1$ to $r = 0.4$. This result is consistent with the observations of $n = 4$ and $n = -1$ activity during NBI. This mode is the first identification of an island-induced Alfvén eigenmode, or IAE, in an RFP. The double-peak nature of the IAE mode is consistent with the coupling between mode numbers expected from the island-induced helical modulation of the magnetic field (in this case $\delta n = 5$) [24]. The 145 kHz frequency found is consistent with the 140–160 kHz observed on MST for the relevant Alfvén speed, $v_A = 1.75 \times 10^6$ m s$^{-1}$.

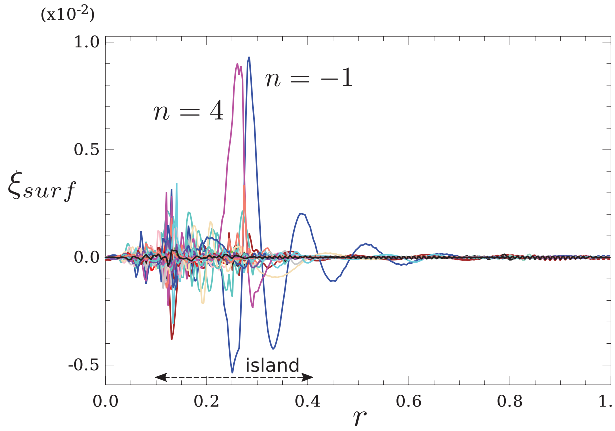


**Figure 5.** Island-induced Alfvén eigenmode (IAE) in MST computed with the SIESTAlfvén code. The mode is dominated by an island-induced helical coupling between the $n = 4$ and $n = -1$ Fourier components. The frequency of the mode is 145 kHz, in agreement with 140–160 kHz from experiment.

## Conclusions

In this work, modifications in the shear Alfvén continuum due to the helical coupling from a magnetic island were used to explain the observations of Alfvén activity in MST. The lowest two continuum branches from the theory were shown to envelope the observed $n = 4$ Alfvénic burst frequencies. With this as motivation, the SIESTAlfvén code was used to identify the island-induced Alfvén eigenmode (IAE) in an MST equilibrium containing a magnetic island. The mode number and frequency of the computed IAE is consistent with observations, resulting in the first identification of an IAE in an RFP.

## Acknowledgments

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