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Fizeau interferometer for measurement of plasma electron current

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A high-resolution, vertically viewing far-infrared polarimeter-interferometer system is currently used on the Madison symmetric torus (MST) reversed-field pinch (RFP) to measure the plasma electron density and *toroidal* current density via Faraday rotation. In this article, we propose a scheme to measure the well-known Fizeau effect, whereby through modest modification of the existing apparatus, the line-integrated *poloidal* current density can also be directly measured. This parameter is important, since the RFP toroidal magnetic field is largely determined by currents flowing within the plasma. The Fizeau effect is a phase shift of an electromagnetic wave associated with movement of a dielectric medium. This motion can be related directly to the plasma electron current. Determining the Fizeau effect involves measurement of the phase shift between two collinear, orthogonally polarized, counterpropagating laser beams. Estimates indicate a phase shift of ~2° is expected for typical MST parameters, well within the existing system resolution. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1780771]

I. INTRODUCTION

On the Madison symmetric torus (MST) reversed-field pinch (RFP), a multichannel, vertically viewing, far-infrared (FIR) laser system has been used as a Faraday rotation polarimeter to measure poloidal magnetic field, and as an interferometer to measure electron density. Recent advances in Faraday rotation diagnostics have provided the first measurement of toroidal current-density profile dynamics in the core of a high-temperature RFP plasma.[1] This technique involves measuring the difference in the refractive index between the right-hand (R-) and left-hand (L-) circularly polarized waves. For the RFP, as well as some tokamaks (e.g., spherical tokamaks), the toroidal magnetic field is also a relative unknown and must be measured. For instance, on the NSTX spherical tokamak, a tangentially viewing Faraday system is being developed to measure the toroidal magnetic field.[2] On MST, this could be accomplished by simply rotating the existing geometry and constructing a tangentially viewing Faraday system. However, such an approach requires significant additional machine access, as well as a separate laser-receiver system. This naturally leads to substantial costs as well. Another approach would be to take advantage of the existing, vertically viewing system and machine access to measure the Cotton-Mouton effect. This results from the linear birefringence of the plasma and is related to the difference in index of refraction between the ordinary (O) and extraordinary (X) waves. On the Wendlestein VII-AS stellarator, this measurement was accomplished and used to determine the electron density in a plasma where the toroidal field was known.[3] A third approach involves measurement of the well known Fizeau effect.[4] For a vertically viewing system, this effect can be used to measure the line-integrated *poloidal* electron current density from which the toroidal field can be derived through Ampere's law. Experimental determination of the poloidal electron current density is extremely important for the RFP, as the toroidal field is largely determined by currents flowing within the plasma (not external coils). In this article, we will describe the Fizeau effect and a means whereby it can be directly measured in MST.

II. FIZEAU EFFECT

It has been known for over a century that the velocity of light in a moving medium differs from its value in a stationary medium.[4] The Fizeau effect is the relativistic phase shift of an electromagnetic wave associated with movement of a dielectric medium. Consider a laser beam propagating in a moving plasma with refractivity index N', length L', and velocity v, as shown in Fig. 1. The phase shift is relativistically invariant and to the lowest order in $\beta = v/c$, the single pass shift measured by an interferometer is

$$\Delta\phi = \Delta\phi' = \omega'\Delta t' - k'\Delta x' = -\beta\frac{\omega'}{c}L' - \frac{\omega'}{c}L'N', \qquad (1)$$

where primed and unprimed variables refer to the moving plasma and laboratory frames, respectively. In the above relation (ω', k') are the frequency and wave vector of the probing lightwave in the plasma. By using the Lorentz transformation and considering a small Doppler shift $(\omega' = \omega - \beta\omega)$, the phase shift in the laboratory frame[5] is given by

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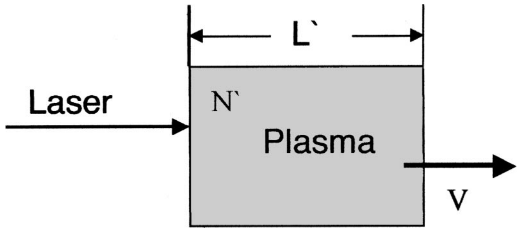

FIG. 1. Schematic diagram of the Fizeau effect.

$$\Delta\phi \approx -\frac{\omega}{c}L'N - \beta\frac{\omega}{c}L'\left(1 - N - \omega\frac{\partial}{\partial\omega}N\right). \qquad (2)$$

The first term on the right-hand side (RHS) of Eq. (2) is the phase shift, $\Delta\phi_0$, which is normally measured by an interferometer for a stationary plasma. The second term on the RHS of Eq. (2) is the additional phase change caused by plasma motion. The phase difference measured by a Fizeau interferometer (i.e., interferometer with counterpropagating beams following the same optical path) in a plasma using an O-mode wave ($N^2 = 1 - \omega_{pe}^2/\omega^2$) is

$$\delta = \frac{1}{\omega c^2}\int v\omega_{pe}^2\, dx = 1.875 \times 10^{-23}\lambda\int n_e(x)v_x(x)dx, \quad (3)$$

where $\omega_{pe}$is the electron plasma frequency, λ is the laser wavelength, and n_e is the electron density. The product $n_e(x)v_x(x)$ in the integral is just the electron current J_e. For a vertically viewing system, such as that on MST, this corresponds to the poloidal electron current density, $J_{\theta,e}$.

III. EXPERIMENTAL SETUP

An estimate of the Fizeau effect for typical MST plasmas with toroidal current $I_p = 400$ kA and electron density $n_c = 1.5 \times 10^{19}$ m^{-3} is shown in Fig. 2. MST equilibrium reconstruction indicates that a maximum poloidal current density of ~1.5 MA/m^2 will occur at approximately midradius, leading to a Fizeau interferometer phase shift of ~2°. Phase shifts due to plasma density are far larger than shifts due to plasma motion. While the phase is small compared to the equilibrium phase shift for stationary medium (>360°), it is comparable to angles resolved by the equilibrium Faraday rotation measurement.[1]

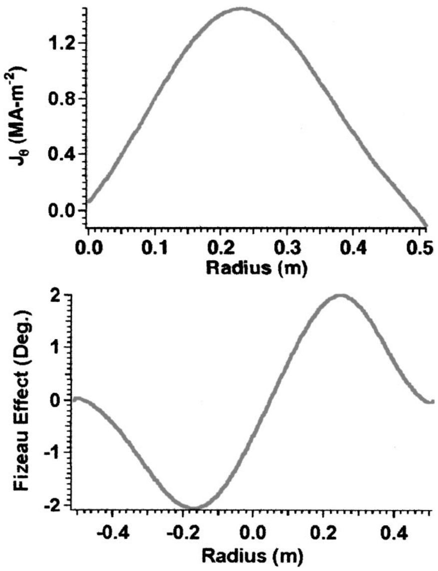

FIG. 2. (a) Poloidal electron current density determined from equilibrium reconstruction. (b) Estimate of the Fizeau phase shift for typical MST plasma.

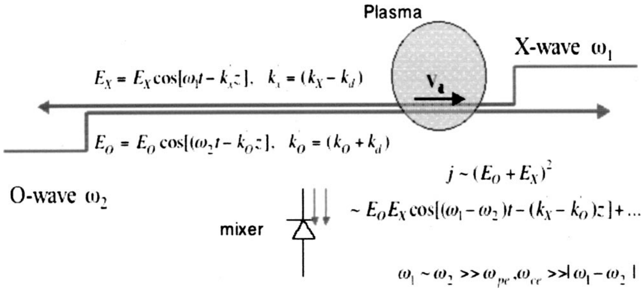

FIG. 3. Schematic of the Fizeau measurement technique.

Due to the small phase angle compared to that induced by the equilibrium density, measurement of the Fizeau effect is impractical for standard interferometers. However, this difficulty can be overcome by measuring the phase shift between two collinear, orthogonally polarized, counterpropagating laser beams. This arrangement is shown schematically in Fig. 3, where the counterpropagating probe beams are the ordinary and extraordinary waves. These waves have slightly different frequencies with $\Delta\omega/2\pi \approx 1$ MHz. The nominal wavelength of the FIR laser is 432 μm (694 GHz). The phase difference between these two waves can be written according to Eq. (3), with the simplification that $k_X - k_O = n_e B_\perp^2 \approx 0$. For MST parameters, the actual value has a maximum of $|k_X - k_O|_{MAX} \approx 0.1°$ and can be directly attributed to the Cotton-Mouton effect mentioned earlier. In this arrangement, zero-order terms cancel out and the contribution due to motion of the medium (Fizeau effect) is doubled. Since the phase resolution of the polarimeter is ~1 mrad (0.06°) with up to ~1 μs time response, it is feasible for this system to also resolve the Fizeau effect.[1,6]

Experimentally, the Fizeau measurement can be accomplished by employing the experimental setup shown in Fig. 4. In this arrangement, one of the laser beams goes through a $\lambda/2$ plate to rotate the polarization 90°. A small portion of

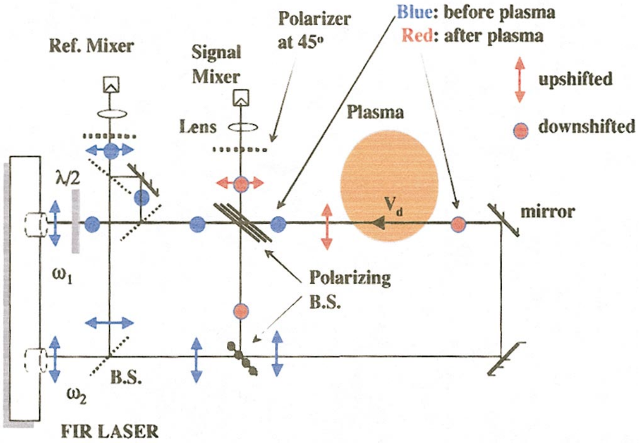

FIG. 4. (Color) Experimental arrangement for measuring the Fizeau effect.

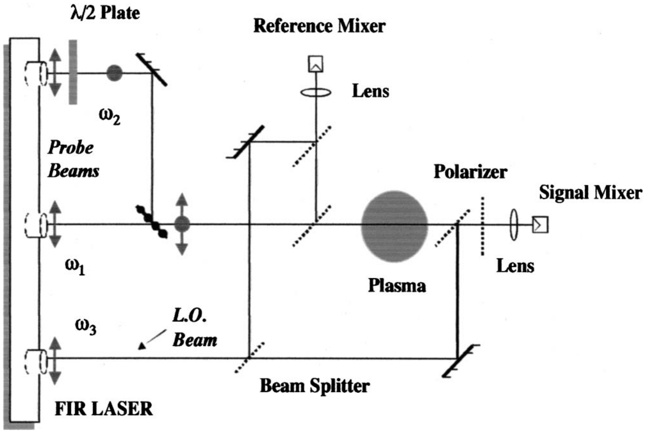

FIG. 5. Experimental arrangement for measuring the Cotton-Mouton effect.

each beam is then taken and mixed in the reference detector. The beam with polarization perpendicular to the page then passes through a polarizing beamsplitter aligned for 100% transmission. This beam proceeds through the plasma, experiences a phase shift due to the equilibrium density and moving medium, and is reflected by a polarizing beamsplitter aligned to reflect 100%. The beam with polarization in the page passes through this beamsplitter (100% transmission) and the plasma, whereupon it is reflected from the polarizing beamsplitter in its path. This beam also sees the equilibrium density, but the sign of the moving medium contribution is reversed. The two beams that passed though the plasma are now combined and sent to the signal mixer. Since both beams see the same equilibrium plasma with the same optical path, this effect cancels out. We are then left with twice the phase shift caused by the Fizeau effect [Eq. (3)]. We then only need to measure the phase difference between the reference and signal mixers to obtain the Fizeau phase shift.

It is worth noting that the Fizeau effect has not been measured in any plasma. We plan to measure this phase shift by far-infrared Fizeau interferometry (FIFI) in order to verify the relationship between phase shift and velocity first [i.e., Eq. (3)]. It is possible, by operating with low discharge current plasmas, to measure the poloidal electron current by using probes or other magnetic-field measurements. Knowing the electron density from standard interferometry will then enable us to calculate the Fizeau phase shift, which can then be directly compared to the phase shift measured by the Fizeau interferometer. Once the plasma Fizeau effect has been verified, we will proceed to measure the electron current density in higher temperature plasmas.

By altering the experimental arrangement as shown in Fig. 5, we can make the two beams collinear but *not* counterpropating. In this instance, the Fizeau effect will cancel out and we are left with a measurement of the phase difference between the O and X waves, or the Cotton-Mouton effect. Since, as mentioned earlier, the Cotton-Mouton effect is expected to be an order of magnitude smaller than the Fizeau effect, it is not clear that we can resolve this phase shift. However, it is clear that by having the beams copropagate, the Fizeau effect should cancel out. We plan to use this as a cross-check against the actual Fizean measurement. In addition, we will be able to perform these measurements for various chords looking in different regions of the plasma. A chord through the midradius should see a maximum Fizeau effect, while a chord through the magnetic axis is expected to have no Fizeau effect. All these tests, and others, can be performed to cross-check the measurement.

In summary, by reconfiguring the existing, vertically viewing Faraday rotation diagnostic on MST, we plan to measure the Fizeau, as well as the Cotton-Mouton effects. Each of these measurements will give us information on the toroidal magnetic field in the RFP. By using the Fizeau effect, we will have a direct measure of the poloidal electron current density. Since the time response and phase resolution of the existing Faraday rotation diagnostic is sufficient to measure both magnetic-field and current-density fluctuations,[6] it is anticipated that the Fizeau interferometer may be able to directly measure fluctuations in the electron current density. If successful, this information in combination with polarimetry can be used to measure the magnetic fluctuation-induced particle flux for the electrons, $\Gamma_e = \langle \tilde{j}_{ll\ e} \tilde{b}_r \rangle$. In addition, if $\tilde{J}_e$ is known from the Fizean measurement and $\tilde{n}_e$ from interferometry, it also becomes feasible to directly determine electron velocity fluctuations, $\tilde{v}_e$.

Finally, application of the Fizeau interferometer measurement on a tangentially viewing system (e.g., such as those on, or proposed for, tokamaks) could be exploited to provide a direct measure of the toroidal current density. This may be particularly relevant to future devices, such as ITER, where access is limited and determination of the toroidal current density is a critical measurement need.

ACKNOWLEDGMENT

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